A Technical Description of the STARS Efficiency Rating System Calculation

The following is a technical description of the efficiency rating calculation process used by the Office of Superintendent of Public Instruction’s (OSPI) Student Transportation Allocation Reporting System (STARS). It is not designed for the general population, but for those mathematically inclined individuals wishing more detailed technical information regarding the calculation process used to develop the efficiency ratings and cohort district weightings. The STARS efficiency system was developed as an integral part of the funding model developed by Management Partnership Services (MPS) under contract to the Office of Financial Management (OFM) and this document was primarily based on MPS’s report to the Washington State Office of Financial Management “Development of Student Transportation Funding Methodology Options for Washington State”, November 21, 2008.

The core of the efficiency rating process is a Data Envelopment Analysis (DEA) calculation. DEA is an established and widely accepted statistical process. To build a properly constructed DEA model, one first identifies the inputs (the resources used), the outputs (the services provided), and the site characteristics (the factors that influence costs, but are beyond the direct control of the school district). The DEA model identifies, for each school district, an empirically based and mathematically sound minimum expenditure level that allows the district to provide transportation services, while recognizing each district’s local site characteristics.

In the STARS process, there are two inputs: operating expenditures and buses; and two outputs: the number of basic and special program students transported. At this point, OSPI has identified the following six school district features that are used as site characteristics: land area, average distance from stops to school, the number of destinations, students per mile of roadway (used as a measure of student density) and roadway miles per land area (used as a measure of roadway density). In the March 2013 and March 2014 ratings, the number of kindergarten routes was included. However, the collection of that data was discontinued in the 2013–14 school year.

General Description

The key concept of the DEA model is the identification for each district of a target district, which is a weighted average of all the school districts in Washington State. DEA uses the optimization power of linear programming to identify, for each school district, how much weight to place on every other district to produce a target district that simultaneously reduces expenditures and buses by the largest possible percentage, while maintaining the number of students transported. The target district does so while operating under the same or worse site characteristics.
As a simplified example, suppose that the linear program for District A identifies its target district to be 60% of District B, 30% of District C, and 10% of District D and that land area is the only site characteristic. Then the expenditures, buses, regular education riders, special education riders, and land area of District A’s target district equal 60% of the value at District B, plus 30% of the value at District C, plus 10% of the value at District D. The table below shows hypothetical data for Districts A through D and for District A’s target.

<table>
<thead>
<tr>
<th>District</th>
<th>Weight</th>
<th>Expenditures</th>
<th>Buses</th>
<th>Basic Riders</th>
<th>Special Riders</th>
<th>Land Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>---</td>
<td>$900,000</td>
<td>32</td>
<td>1530</td>
<td>191</td>
<td>130</td>
</tr>
<tr>
<td>B</td>
<td>60%</td>
<td>$1,000,000</td>
<td>30</td>
<td>2000</td>
<td>250</td>
<td>200</td>
</tr>
<tr>
<td>C</td>
<td>30%</td>
<td>$100,000</td>
<td>10</td>
<td>100</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>D</td>
<td>10%</td>
<td>$2,000,000</td>
<td>60</td>
<td>3000</td>
<td>380</td>
<td>100</td>
</tr>
<tr>
<td>Target</td>
<td></td>
<td>$830,000</td>
<td>27</td>
<td>1530</td>
<td>191</td>
<td>139</td>
</tr>
</tbody>
</table>

The expenditures for District A’s target would be \((0.6) \times ($1,000,000) + (0.3) \times ($100,000) + (0.1) \times ($2,000,000) = $600,000 + $30,000 + $200,000 = $830,000\). The same calculation would be performed for buses, basic program riders, special program riders, and land area. The important observation is that District A’s target performs better than District A. The target spends $70,000 less, uses 5 fewer buses, and transports the same number of students, while it has more land area, a factor known to increase cost. Yet, it is reasonable to assume that the performance of the target district is achievable since it is a weighted average of actual districts (if land area was the determining characteristic of school district transportation performance). Note that if District A were operating efficiently, then it would have placed 100% of its weight on itself and its target would be identical to District A.

**Details of the Calculation Process**

Let \(n\) be the number of school districts. The DEA literature would refer to school districts as a service delivery unit or decision-making unit (DMU). For this discussion, SD will be used to indicate the school district as the unit under consideration. Let \(X_{ij}\) be amount of input \(i\) consumed by SD \(j\), for \(i = 1, 2, \ldots, I\) and \(j = 1, 2, \ldots, n\). In STARS, where the inputs are prior year expenditures and buses, \(X_{1j}\) would be the prior year expenditures for SD \(j\) and \(X_{2j}\) would be the number of buses.

Let \(Y_{rj}\) be the amount of output \(r\) produced by SD \(j\), for \(r = 1, 2, \ldots, R\) and \(j = 1, 2, \ldots, n\). The outputs represent the “production levels” of the DMUs. In STARS, the outputs are the basic program student count \((Y_{1j})\) and the special program student count \((Y_{2j})\).

In DEA theory, while the choices of inputs and outputs must capture the essence of each DMU’s productive operations, it is often difficult to describe completely all the
inputs that a DMU consumes and all the outputs that it produces. As a result, every effort must be taken to define the inputs and outputs as fully as possible. Fortunately, in the evaluation of school transportation, the inputs and outputs are fairly straightforward.

Let $S_{kj}$ be the value of site characteristic $k$ at SD $j$, for $k = 1, 2, \ldots, K$ and $j = 1, 2, \ldots, n$. A site characteristic describes a feature of a school district that influences its ability, favorably or unfavorably, to transport students per dollar of expenditure. For example, the number of roadway miles per square mile is a favorable site characteristic because a larger value indicates that a school district is likely to spend less money and use fewer buses to deliver given numbers of basic and special program students to school.

Two questions need to be answered with respect to each site characteristic. First, does it belong in the model? There may be reasons to suspect that it has an influence on the ability of a school district to operate efficiently, but it may not be obvious that the effect is real. Second, if the effect is real, is the site characteristic favorable or unfavorable? In some cases, there is little question about the favorable or unfavorable nature of the site characteristic, but in other cases it may not be clear.

The STARS process addresses both questions by constructing a multiple regression model using operating expenditures as the dependent variable and using all of the outputs and all of the site characteristics as potential independent variables. STARS uses operating expenses as the dependent variable, because the formula focuses on reducing the total cost of operations. Some data elements have been converted using the natural logarithm to ensure that all the standard regression model assumptions are satisfied. The site characteristics that remain in the regression model are used in the efficiency rating system and the signs of the coefficients reveal the nature of the site characteristics. If its coefficient is positive, then the site characteristic is unfavorable (higher values of the site characteristic are associated with higher operating costs) and if its coefficient is negative, then the site characteristic is favorable (higher values of the site characteristic are associated with lower operating costs).

A quality measure represents the degree of excellence or complexity associated with the overall performance of the SD. The STARS model currently does not incorporate any measures of quality or safety, due to lack of appropriate statewide data. However, it is possible that future versions of the model will include quality measures, such as average time spent on the bus, or appropriate safety measures such as out-of-service school bus inspections. In that case, $Q_{mj}$ would be the value of quality measure $m$ at SD $j$, for $m = 1, 2, \ldots, M$ and $j = 1, 2, \ldots, n$.

An input orientation is used because the objective is to find the lowest possible cost for delivering the SD’s outputs. A variable returns to scale model is used in many DEA applications where the DMUs display a wide variation in sizes. A variable returns to
scale model helps to reduce the bias that may be exhibited by a constant returns to scale model. A variable returns to scale model is appropriate, since the school districts in Washington display significant variations in size.

The problem is then to solve an input-oriented DEA model with variable returns to scale, which requires the solution of one linear program for each SD. The linear program for SD \( d \), \( d = 1, 2, \ldots, n \), is:

Min \( E_d \) \hspace{1cm} (1)

subject to

\[ \sum_{j=1}^{n} \lambda_j X_{ij} \leq E_d X_{id} \text{ for } i = 1, 2, \ldots, I \] \hspace{1cm} (2)

\[ \sum_{j=1}^{n} \lambda_j Y_{rj} \geq Y_{rd} \text{ for } r = 1, 2, \ldots, R \] \hspace{1cm} (3)

\[ \sum_{j=1}^{n} \lambda_j S_{kj} \leq or \geq S_{kd} \text{ for } k = 1, 2, \ldots, K \] \hspace{1cm} (4)

\[ \sum_{j=1}^{n} \lambda_j Q_{mj} \leq or \geq Q_{md} \text{ for } m = 1, 2, \ldots, M \] \hspace{1cm} (5)

\[ \sum_{j=1}^{n} \lambda_j = 1 \] \hspace{1cm} (6)

\[ \lambda_j \geq 0 \text{ for } j = 1, 2, \ldots, n \] \hspace{1cm} (7)

\[ E_d \geq 0 \] \hspace{1cm} (8)

Note that setting \( \lambda_d = 1, \lambda_j = 0 \text{ for } j \neq d \), and \( E_d = 1 \) is a feasible, but not necessarily optimal, solution to the linear program for SD \( d \). This implies that \( E_d^* \), the optimal value of \( E_d \), must be less than or equal to 1. The optimal value, \( E_d^* \), is the overall efficiency of SD \( j \). The left-hand-sides of Equations (2)-(5) are weighted averages, because of Equation (6), of the inputs, outputs, site characteristics, and quality measures, respectively, of the \( n \) SDs. At optimality, that is with the \( \lambda_j \) replaced by \( \lambda_j^* \), the left-hand-sides of Equations (2)-(5) would be called the target inputs, target outputs, target site characteristics, and the target quality measures, respectively, for SD \( d \).

Equation (2) implies that each target input will be less than or equal to the actual level of that input at SD \( d \). Similarly, Equation (3) implies that each target output will be greater than or equal to the actual level of that output at SD \( d \).

The nature of each site characteristic inequality in Equation (4) depends on the manner in which the site characteristic influences efficiency. For a favorable site characteristic (larger values imply higher efficiency, on average), the less-than-or-equal to sign is used, while for an unfavorable site characteristic (larger values imply lower efficiency,
on average), the greater-than-or-equal to sign is used. In the future, a site characteristic could be included using a 0-1 indicator variable to reflect membership in a category, such as "operating in a rural area". If this category were used, when analyzing a school district operating in a rural area, only other districts operating in rural areas would be allowed to appear with positive weight in the target SD. In such cases, the equal sign is used. Thus, Equation (4) implies that the value of each target site characteristic will be the same as or worse than the actual value of that site characteristic at SD \( d \).

Similarly, the nature of any quality measure inequality in Equation (5) depends on the scale of the quality measure. If a quality measure is such that larger values represent higher quality, then the greater-than-or-equal-to sign is chosen, while if the quality measure is such that smaller values represent higher quality, the choice would be the less-than-or-equal-to sign. Thus, Equation (5) implies that the value of each target quality measure will be the same as or better than the actual value of that quality measure at SD \( d \).

As a result, the optimal solution to the linear program for SD \( d \) identifies a hypothetical target SD \( d^* \) that, relative to SD \( d \); consumes the same or less of every input (uses less money or buses), produces the same or more of every output (transports as many or more students), operates under the same or worse site characteristics, and achieves quality measures that are the same or better. Moreover, the objective function expressed in Equation (1) ensures that the target SD \( d^* \) consumes input levels (expenditures and buses) that are reduced as much as possible in across-the-board percentage terms.

An essential premise is that a SD could in fact operate exactly as does SD \( d^* \). In the theory of production, this is the assumption, made universally by economists, that the production possibility set is convex. In this context, the production possibility set is the set of all vectors \( \{X_i, Y_i, S_k, Q_m\} \) of inputs, outputs, site characteristics, and quality measures such that it is possible for a SD to use input levels \( X_i \) to produce output levels \( Y_i \) under site characteristics \( S_k \) while achieving quality measures \( Q_m \). The convexity assumption assures that SD \( d^* \) is feasible and that it is reasonable to expect that SD \( d \) could modify its performance to match the performance of \( d^* \).

**Applying the DEA Process in Washington State**

In the 2011-12 academic year, the 288 school districts providing transportation services in Washington State incurred $405,623,489 in operating expenditures and used 7,455 school buses to transport 355,051 basic program students and 25,182 special program students to and from school. These counts are pro-rated from the Fall, Winter and Spring reports from the 2011-12 school year. Thus, the DEA model has \( n = 288 \) DMUs, \( I = 2 \) inputs (each district's operating expenditures and buses), and \( R = 2 \) outputs (each
district’s basic program student count and special program student count). There are $K = 5$ site characteristics identified using a regression model that used the natural logarithm of operating expenditures as the dependent variable.

The following table shows the five site characteristics that were statistically significant in this process and whether higher values of each site characteristic are associated with more or less favorable operating conditions. Also included as site characteristics are four binary variables, each with its own constraint, that indicate the percentile grouping of the school district with respect to total students transported. The equal sign is used as the constraints for the quartile site characteristics, which implies that a school district can only place positive weight, $\lambda^*$, on other school districts in its own quartile.

<table>
<thead>
<tr>
<th>Site Characteristic</th>
<th>Favorable/Unfavorable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Land Area</td>
<td>Unfavorable</td>
</tr>
<tr>
<td>Average Student Distance to School</td>
<td>Unfavorable</td>
</tr>
<tr>
<td>Number of Locations Served</td>
<td>Unfavorable</td>
</tr>
<tr>
<td>Road Miles per Square Mile of Land Area</td>
<td>Favorable</td>
</tr>
<tr>
<td>Students per Roadway Mile of Land Area</td>
<td>Favorable</td>
</tr>
</tbody>
</table>

To perform these calculations, OSPI uses a macro developed by MPS written in Visual Basic for Applications® (VBA) to solve the $n$ linear programs sequentially and save the results in a spreadsheet. The VBA uses the Solver® add-in (Frontline Systems, Inc., Incline Village, NV) in Microsoft Excel® to solve the linear programs. While both Solver and VBA are available in all versions of Excel, the standard version of Solver is limited to 200 variables and 200 constraints, which limits the size of the problems to no more than 199 school districts and no more than 199 inputs, outputs, site characteristics, and quality measures, combined. As a result, OSPI uses Premium Solver, available for purchase from Frontline Systems, Inc.

The VBA program produces several worksheets and charts that summarize the results of the DEA. One worksheet shows the overall and factor efficiencies for each district along with actual and target values for each input. Another worksheet contains information on the efficient reference set of cohort districts and associated weights for each district. This worksheet is imported into STARS. The software also provides statewide frequency tables and charts.

**Conclusion**

While the creation and implementation of an efficiency rating system is challenging, the rewards can be substantial. Besides the measurable benefit of lowered costs, an efficiency rating process may alter the manner in which school districts make important strategic and tactical decisions. An efficiency rating system should lead to more
efficiently managed school district transportation operations. By contrast, the STARS funding approach is based on expected costs and will tend to lead to average, rather than best, performance.

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