

Test and Item Specifications

Integrated Mathematics 2



The purpose of the end-of-course exams is to measure the level of mathematics proficiency that Washington students have achieved based on Washington State K-12 Mathematics Standards. In the 2008 revision, the Washington State K-12 Mathematics Standards are organized by areas of emphasis as: Core Content, Additional Key Content and Core Processes. Each area of emphasis has specific performance expectations.

Test and Item Specifications Integrated Mathematics 2

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Test Development Guidelines Integrated Mathematics 2

The items on the Integrated Mathematics 2 End-of-Course Exam reflect the performance expectations of the Washington State Mathematics Learning Standards. The guidelines in this document assist in writing items that match the performance expectations and their associated restrictions. Restrictions are necessary to construct a valid and reliable on-demand assessment. These restrictions are not necessary in classroom based assessments.

The item writer should be familiar with all item and rubric development guidelines listed in this section as well as specific considerations listed within each area of emphasis. A style guide determines the format of items and will be applied after the items are written.

Considerations and procedures that make item development more efficient and effective include, but are not limited to, the following guidelines.

Item Development Guidelines Standards/Performance Expectations

- Students are expected to know all content, vocabulary, and processes in grades 3-8 and Algebra 1/Integrated Mathematics 1.
- An item may assess all or part of a performance expectation.
- An item may assess one or more performance expectations within a single area of emphasis.
- An item assessing Core Processes will use mathematics from the Core Content or Additional Key Content at the grade level being assessed.
- When applicable use stem, stimulus, and prompt rules for specific performance expectations found in each area of emphasis.

General Considerations

- Any contexts used in an item should be familiar to students.
- Stimulus content will be factually correct.
- Focus on what is essential and consequential to minimize the impact of, or need for, outside knowledge.
- Stimulus should not “trick” students into choosing or developing an incorrect or ineffective response.
- Items should avoid use of “not” or “if” unless it is essential to communicate understanding of the task. Consider substituting “when” for “if”.
- Items will include language that is unbiased and that will not disadvantage a particular group of students.
- Items do not contain language or representations that might offend or demean any group of students.
- Character names will be assigned from a database that is representative of the range of ethnic diversity of Washington students.

- Manipulatives are not necessary for the exam but may be made available in the classroom.

Manipulatives and Tools Allowed	Manipulatives and Tools Not Allowed
<p>Manipulatives that are used during the assessment should not be distributed to the students but should be available in the classroom to students who elect to use them.</p> <ul style="list-style-type: none"> • Straightedge (all grades) • Ruler with centimeters and inches for grade 3 (required) • Protractor or angle ruler for grade 5 (required) • Compass for Year 2 EOC only • Abacus for visually impaired/blind students using Braille edition • Tiles, algebra tiles, cubes • Base-ten pieces • Pattern blocks, geoboards, Cuisenaire rods • Judy clocks without a digital display • Glossary of Non-Mathematics Terms • Graph paper for grades 3-8 only (must be collected and shredded) <p><i>Tools that can remain on teachers' walls:</i></p> <ul style="list-style-type: none"> • Hundreds charts (0-99 or 1-100 only) • Number lines with whole numbers only 	<p>Because of the multitude and variety of materials available, the following list of materials that are <u>not</u> allowed is not exhaustive. Consider all manipulatives “Not Allowed” if they are not listed as “Allowed”.</p> <p>The following list addresses the most commonly asked questions concerning manipulative use from the field.</p> <ul style="list-style-type: none"> • Calculators for grades 3-6 • Multiplication or addition matrices • Number lines with integers, fractions, decimals, or markings of multiples, prime, and/or composite numbers • Commercially- or student-made fraction pieces, fraction templates, or fraction materials, whether labeled or unlabeled • Dictionaries or thesauruses • Patty paper or tracing paper • Dry erase boards • Highlighters

- Calculators are allowed on the grade 7, 8, and end-of-course only. Please see the calculator policy at <http://www.k12.wa.us/Mathematics/CalculatorPolicy.aspx>.

Vocabulary/Context

Clear Language

- Item stems and stimulus materials should be straightforward and use simple syntax.
- Stimulus should be clear and simple with a minimum of distracting or irrelevant information unless it is appropriate for the performance expectation being assessed.
- The amount of reading should be kept to a minimum so that each item is clear and precise.
- Items will clearly indicate what is expected in a response to help students focus their responses.

Vocabulary

- Use vocabulary excel sheet located at <http://www.k12.wa.us/Mathematics/TestItemSpec.aspx>.
- Items use language targeted to the previous grade level or lower readability, except for required mathematics terms listed in the Test and Item Specifications document.

- Items will not assess vocabulary definitions or theorems directly, but will assess conceptual understanding and application.

Notational Considerations

- Numbers, other than years, having more than three digits to the left of the decimal point will include commas to group digits as in 435,000.
- Standard measurement abbreviations may be used; however, the unit should be spelled out if any confusion is possible, e.g., “inch” rather than “in.”
- Letters used as variables are always italicized.
- The symbols “ \times ” and “ \bullet ” may be used to indicate multiplication.
- Parentheses or brackets may be used as grouping symbols to indicate multiplication.
- The symbol “ \div ” or a horizontal fraction bar may be used to indicate division.
- Fractions will have a horizontal line separating numerator and denominator, e.g., $\frac{1}{2}$.
- Large numbers may be represented with a heading labeled “in thousands” or “in millions” in tables, charts, or graphs.
- Decimals between negative one and one are written with a leading zero, e.g., 0.25 rather than .25.
- Illustrations of figures may include hash marks on line segments to indicate congruent sides.
- Right angles will be indicated in the graphics or item stem.
- The symbol “ \perp ” may be used to indicate perpendicular lines, e.g., $\ell_1 \perp \ell_2$ at grade 8 MSP and end-of-course exams.
- The symbol “ \parallel ” may be used to indicate parallel lines, e.g., $\ell_1 \parallel \ell_2$ at grade 8 MSP and end-of-course exams.
- Each graph or table will include a title in the prompt and/or the answer space.

Rules for Stimulus, Stem, and Prompt Content

- Stimulus content should be clear and simple with a minimum of distracting or irrelevant information unless it is appropriate for the performance expectation being assessed.
- Stimulus content should not “trick” students into choosing or developing an incorrect or ineffective response.
- Stimulus may include appropriate and relevant tables, charts, graphs, diagrams, and/or pictorial representations of objects, shapes, or figures.
- Items will focus on what is essential and consequential to minimize the impact of, or need for, outside knowledge.
- The amount of reading will be kept to a minimum so that each item is clear and precise.

Rules for Multiple-Choice Items

- Each Multiple-Choice item has four answer choices, the correct answer and three distractors (wrong answer choices).
- Each Multiple-Choice item will have a stem (question or statement).
- Multiple-Choice item stems will present a clear indication of what is required so that

students will know what to do before looking at the answer choices.

- The four answer choices will be approximately the same length, will have the same format, and will be syntactically and semantically parallel.
- The answer choices will be arranged in numerical or chronological order or according to length.
- Students should not be able to rule out a distractor or identify the answer simply because of superficial or trivial characteristics, syntactic complexity, or concept complexity.
- Distractors will reflect common errors or misunderstandings, naive pre-conceptions, or other misconceptions.
- Distractors will not be partially correct responses nor will they be designed to “trick” students into responding incorrectly.
- The responses "All of the above" and "None of the above" will not be used.
- The letters A, B, C, and D will be used for answer choices and will not be used as labels within a multiple-choice item in either upper or lower case.

Rules for Completion Items

- Completion items should be written like a multiple-choice item but no answer choices are provided.
- Completion items will give clear indications of what is required of students.
- Completion items will have a unique numeric answer or a number in a specified interval, i.e. answers derived using estimates of pi.
- Completion items will give a directive and reword the directive in the format of a question in the box with a line for the student’s response. e.g. “Determine the quotient.”, “What is the quotient?”
- Answer will not be scored for labels. Labels should be included in the question and/or answer space.

Rules for Short-Answer Items

- All EOC Short-Answer items assess content PEs in reporting strands used to calculate student scores for purposes of meeting standard. Please refer to EOC Test Specifications for further information.
- Short-Answer items will give clear indications of what is required of students; e.g., “Name two properties of Figure A.” or “Write an equation.”
- Anything required by the scoring rubric will be asked for in the item.
- Item response spaces may be written to guide responses. A response that requires multiple parts may be scaffolded with boxes to draw attention to the parts.
- Directions with multiple requirements will be organized with bullets.
- When an item poses a specific question, the question is repeated at the bottom of the workspace with a line for the student’s response.
- When an item gives a directive, then a question based on the directive will appear in the box with a line for the student’s response. e.g. “Determine the surface area of the pyramid.”, “What is the surface area of the pyramid?”

- General directions that allow the student to construct a response may read as follows: "Show your work using words, numbers and/or pictures." "Show the steps you used to solve the problem."
- Any Short-Answer item that requires the student to use information from a stimulus will specifically ask for the information; e.g., "Use numbers from the table to ..." or "Support your answer with information from the chart."
- Short-Answer items may ask for a figure, diagram, equation, and/or a few sentences.
- Short Answer items will require a limited number of steps to develop a viable solution, demonstrate an understanding or process, communicate a mathematical idea or result, or show reasoning.
- Short-Answer items include Enhanced Multiple-Choice items that ask students to select from a list of four answer choices and then show work to support or explain the reason(s) for choosing that response.

Rules for Process Items

- The process PEs require a connecting content PE; e.g., 6.6.C (6.1.D). The 6.6.C indicates the item presents a sixth-grade problem-solving situation. The (6.1.D) indicates that multiplication and division of non-negative fractions is likely to be needed to solve the problem. EOC example of process PE; e.g., A1.8.B(A1.3.A)/M1.8.B(M1.2.A). The A1.8.B/M1.8.B indicates the item present an Algebra 1/Integrated Mathematics 1 problem solving situation. The A1.3.A/M1.2.A indicates that the content relates to functions and their characteristics.

Scoring Rubric Development Guidelines

- An item-specific scoring rubric will be developed for each Short-Answer item during the writing of the item.
- Short-Answer items will be scored with a 3-level scoring rubric (0-2).
- Score point elements will be based on the requirements of the item and its performance expectation(s).
- Scoring rubrics will focus on conceptual understanding, application of appropriate procedures/strategies, and accuracy.
- Scoring rubrics will not consider conventions of writing (complete sentences, usage/grammar, spelling, capitals, punctuation, and paragraphing), as long as the wording of the response does not interfere with the mathematical communication.
- Scoring rubrics that involve measured values will require students to label units when the labels are not provided in the answer space.
- Scoring rubrics will be edited during pilot range finding.
- Scoring rubrics may be edited during operational range finding.

Cognitive Complexity

from Depth-of-Knowledge Levels for Four Content Areas, Norman L. Webb, March 28, 2002

“Level 1 (Recall) includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, and straight algorithmic procedure should be included at this lowest level. Other key words that signify a Level 1 include “identify,” “recall,” “recognize,” “use,” and “measure.” Verbs such as “describe” and “explain” could be classified at different levels depending on what is to be described and explained.

“Level 2 (Skill/Concept) includes the engagement of some mental processing beyond a habitual response. A Level 2 assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords that generally distinguish a Level 2 item include “classify,” “organize,” “estimate,” “make observations,” “collect and display data,” and “compare data.” These actions imply more than one step. ... Caution is warranted in interpreting Level 2 as only skills because some reviewers will interpret skills very narrowly, as primarily numerical skills, and such interpretation excludes from this level other skills such as visualization skills and probability skills, which may be more complex simply because they are less common. Other Level 2 activities include explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

“Level 3 (Strategic Thinking) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity, however, that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and using concepts to solve problems.

“Level 4 (Extended Thinking) requires complex reasoning, planning, developing, and thinking most likely over an extended period of time.” (This level is best assessed at the classroom level and not relevant to this on-demand assessment.)

Mathematics 2

M2.1. Core Content: *Modeling situations and solving problems (Algebra)*

This first core content area highlights the types of problems students will be able to solve by the end of Mathematics 2. Students extend their ability to model situations and solve problems with additional functions and equations in this course. Additionally, they deepen their understanding and proficiency with functions they encountered in Mathematics 1 and use these functions to solve more complex problems. When presented with a word problem, students determine which function or equation models the problem and then use that information to write an equation to solve the problem. Turning word problems into equations that can be solved is a skill students hone throughout the course.

M2.2. Core Content: *Quadratic functions, equations, and relationships (Algebra)*

Students learn that exponential and quadratic functions can be used to model some situations where linear functions may not be the best model. They use graphical and numerical methods with exponential functions of the form $y = ab^x$ and quadratic functions to analyze relationships, represent and model problems, and answer questions. Students extend their algebraic skills and learn various methods of solving quadratic equations over real or complex numbers, including using the quadratic formula, factoring, graphing, and completing the square. They learn to translate between forms of quadratic equations, applying the vertex form to evaluate maximum and minimum values and find symmetry of the graph, and they learn to identify which form should be used in a particular situation. They interpret the meaning of problem solutions and explain their limitations. Students recognize common examples of situations that can be modeled by quadratic functions, such as maximizing area or the height of an object moving under the force of gravity. They compare the characteristics of quadratic functions to those of linear and exponential functions. The understanding of these particular types of functions, together with students' understanding of linear functions, provides students with a powerful set of tools to use mathematical models to deal with problems and situations in advanced mathematics courses, in the workplace, and in everyday life.

M2.3. Core Content: *Conjectures and proofs (Algebra, Geometry/Measurement)*

Students extend their knowledge of two-dimensional geometric figures and their properties to include quadrilaterals and other polygons, with special emphasis on necessary and sufficient conditions for triangle congruence. They work with geometric constructions, using dynamic software as a tool for exploring geometric relationships and formulating conjectures and using compass-and-straightedge and paper-folding constructions as contexts in which students demonstrate their knowledge of geometric relationships. Students define the basic trigonometric ratios and use them to solve problems in a variety of applied situations. They formalize the reasoning skills they have developed in previous grades and solidify their understanding of what it means to mathematically prove a geometric statement. Students encounter the concept of formal proof built on definitions, axioms, and theorems. They use inductive reasoning to test conjectures about geometric relationships and use deductive reasoning to prove or disprove their conclusions. Students defend their reasoning using precise mathematical language and symbols. Finally, they apply their knowledge of linear functions to make and prove conjectures about geometric figures on the coordinate plane.

M2.4. Core Content: Probability *(Data/Statistics/Probability)*

Students formalize their study of probability, computing both combinations and permutations to calculate the likelihood of an outcome in uncertain circumstances and applying the binomial theorem to solve problems. They apply their understanding of probability to a wide range of practical situations, including those involving permutations and combinations. Understanding probability helps students become knowledgeable consumers who make sound decisions about high-risk games, financial issues, etc.

M2.5. Additional Key Content *(Algebra, Measurement)*

Students grow more proficient in their use of algebraic techniques as they use these techniques to write equivalent expressions in various forms. They build on their understanding of computation using arithmetic operations and properties and expand this understanding to include the symbolic language of algebra. Students understand the role of units in measurement, convert among units within and between different measurement systems as needed, and apply what they know to solve problems. They use derived measures such as those used for speed (e.g., feet per second) or determining automobile gas consumption (e.g., miles per gallon).

M2.6. Core Processes: Reasoning, problem solving, and communication

Students formalize the development of reasoning in Mathematics 2 as they use algebra, geometry, and probability to make and defend generalizations. They justify their reasoning with accepted standards of mathematical evidence and proof, using correct mathematical language, terms, and symbols in all situations. They extend the problem-solving practices developed in earlier grades and apply them to more challenging problems, including problems related to mathematical and applied situations. Students formalize a coherent problem-solving process in which they analyze the situation to determine the question(s) to be answered, synthesize given information, and identify implicit and explicit assumptions that have been made. They examine their solution(s) to determine reasonableness, accuracy, and meaning in the context of the original problem. The mathematical thinking, reasoning, and problem-solving processes students learn in high school mathematics can be used throughout their lives as they deal with a world in which an increasing amount of information is presented in quantitative ways and more and more occupations and fields of study rely on mathematics.

Test Organization Integrated Mathematics 2

The Integrated Mathematics 2 End-of-Course Exam and Retake Year 1 will contain 37 items that assess the performance expectations common to Geometry/Integrated Mathematics 2. These items are used to determine a student's scale score for purposes of graduation. The performance expectations common to Geometry/Integrated Mathematics 2 will be assessed using multiple-choice, completion, and short-answer items on the end-of-course exams and retake exams.

In addition, the Integrated Mathematics End-of-Course Exam will contain 6 items that assess the performance expectations common to Algebra 1/Integrated Mathematics 2 and Algebra 2/Integrated Mathematics 2. These performance expectations are assessed and reported but are not incorporated into a student's scale score for purposes of graduation. These performance expectations will be assessed using multiple-choice and completion items on end-of-course exams only.

Multiple-Choice Items	Completion Items	Short-Answer Items
<ul style="list-style-type: none"> • Each Multiple-Choice item has four answer choices, the correct answer and three distractors. • Multiple choice items are worth one point each. • There will be 29 Multiple-Choice items assessing PEs common to Geometry/Integrated Mathematics 2. • There will be 3-5 Multiple-Choice items assessing PEs common to Algebra 1/Integrated Mathematics 2 and Algebra 2/Integrated Mathematics 2. <p>NOTE: Enhanced Multiple-Choice items are scored as Short-Answer items.</p>	<ul style="list-style-type: none"> • Each Completion item requires the student to enter a numerical answer, an expression with variables, or an equation with variables. • Completion items are worth one point each. • There will be 5 Completion items assessing PEs common to Geometry/Integrated Mathematics 2. • There will be 1-3 Completion items assessing PEs common to Algebra 1/Integrated Mathematics 2 and Algebra 2/Integrated Mathematics 2. 	<ul style="list-style-type: none"> • Each Short-Answer item requires a constructed response. • A Short-Answer item may ask the student to write a sentence or equation; complete a table, graph, or chart; draw a picture; construct a diagram; or perform a calculation. • An Enhanced Multiple-Choice item will ask the student to select from a list of four answer choices and then show work to support or explain the reason(s) for choosing that answer or to solve a problem. No more than two items on a test will be Enhanced Multiple-Choice items. • Short-Answer items are worth two points each. • There will be 3 Short-Answer items assessing PEs common to Geometry/Integrated Mathematics 2. • Short-Answer items will not assess PEs common to Algebra 1/Integrated Mathematics 2 or Algebra 2/Integrated Mathematics 2.

The Integrated Mathematics 2 End-of-Course Exam is intended to be administered to intact classrooms over three days.

OSPI’s recommendations on administering the EOCs are as follows:

1. The EOCs should be given to intact classrooms over three days (30 minutes for directions/distributing materials, 120 minutes of testing time).
2. Students are expected to finish within 120 minutes of testing time.

Tools, including approved calculators, are allowed for the test administration. Please refer to the calculator policy for more information.

Integrated Mathematics 2 End-of-Course Test Map

Reporting Strands	Number of Items				Total Number of Points
	MC	CP	SA	Total	
Logical arguments and proof*	5-8	0	0-1	6-8	6-8
Proving and applying properties of 2-dimensional figures *	15-19	2-4	1-3	21-24	24-26
Figures in a coordinate plane and measurement *	5-8	1-3	0-1	7-9	7-9
Total Number of Items used to determine scale score**	29	5	3	37	
Total Number of Points used to determine scale score**	29	5	6		40
Course-Specific content***	3-5	1-3	0	6	6
Total Number of Items				43	

**Items assessing these reporting strands assess performance expectations common to Geometry/Integrated Mathematics 2 and are used to calculate student scores for purposes of meeting standard.*

***A scale score on the end-of-course exam is used to determine a student’s proficiency level: below basic, basic, proficient, advanced.*

****Items assessing course-specific content are in addition to the 37 items used to determine a student’s scale score. Student performance on these items is reported but is not incorporated into student scale scores for purposes of meeting standard.*

Integrated Mathematics 2 Performance Expectations by Reporting Strand

Reporting Strands	Performance Expectations	
	Content	Process**
Logical arguments and proof	M2.3.A	M2.6.A
	M2.3.B	M2.6.B
	M2.3.C	M2.6.C
	M2.3.D	M2.6.E
		M2.6.G
Proving and applying properties of 2-dimensional figures	M2.3.E	M2.6.A
	M2.3.F	M2.6.B
	M2.3.G	M2.6.C
	M2.3.H	M2.6.E
	M2.3.I	M2.6.G
	M2.3.J	
	M2.3.K	
Figures in a coordinate plane and measurement	M2.3.L	M2.6.A
	M2.3.M	M2.6.B
	M2.5.B	M2.6.C
	M2.5.C	M2.6.E
		M2.6.G
Course-Specific content	M2.1.A	
	M2.1.B	
	M2.1.C	
	M2.1.D	
	M2.1.E	
	M2.2.A	
	M2.2.B	
	M2.2.C	
	M2.2.D	
	M2.2.E	
	M2.2.F	
	M2.2.H	
	M2.2.G	
	M2.4.A	
	M2.4.B	
	M2.4.C	
	M2.4.D	
	M2.5.A	
	M2.5.D	

*** Items assessing process performance expectations (PEs) must include content at the Integrated Mathematics 2 level. The content of items assessing process PEs determines its reporting strand.*

Item Specifications Integrated Mathematics 2

Item specifications for each Area of Emphasis are organized in two sections:

Stimulus, Stem, and Prompt Rules

Stimulus, stem, and prompt rules list area-specific guidelines for developing items. The rules are in addition to those included in the Item Development Guidelines.

Content Expectations

The performance expectations in this document are identical to those in the Washington State Mathematics 2 Mathematics Standards. Performance expectations that will be assessed at the state level appear in **bold text**. The remaining performance expectations, which appear in *italicized text*, should be taught and assessed at the classroom level.

Items assessing M1.8 Core Processes will use Core Content or Additional Key Content performance expectations from Mathematics 2.

The information in the columns to the right of each performance expectation shows item development parameters for:

- Cognitive complexity (C.C.) as Level 1, Level 2, Level 3, or Level 4 as defined by Norman Webb.
- Format as Multiple-Choice (MC), Completion (CP), or Short-Answer (SA).
- Contextual Situation (Ctxt) as required (Y), item dependent (I), or not allowed (N).

M2.1 Core Content: Modeling situations and solving problems

(Algebra)

Stimulus, Stem, and Prompt Rules

- Use Item Development Guidelines at the beginning of this document.
- Items assessing M2.1 may expect students to describe appropriate domain restrictions.
- Items assessing M2.1 may expect students to use a given function to solve the problem and interpret the solution in the context of the original situation.
- Items assessing M2.1.A may include linear, exponential, or quadratic functions.
- Items assessing M2.1.B may include non-linear equations or inequalities.
- Items assessing M2.1.B-E may expect students to represent a word problem as an equation, inequality, or system of equations or inequalities.
- Items assessing M2.1.D will include functions of the form $y = ab^x$ where b may be less than 1.
- Items assessing M2.1.D may expect students to approximate solutions and, when possible, give answers that are numerically exact.
- Items assessing M2.1.D may ask students to approximate solutions for x in the equation $y = ab^x$.
- Items assessing M2.1.E may expect students to identify a diagram, table, chart, or list of all possible combinations or permutations.

Content Expectations

Items may ask students to:	<i>Traditional Sequence</i>	<i>C.C.</i>	<i>Format</i>	<i>Ctxt</i>
M2.1.A Select and justify functions and equations to model and solve problems.	A1.1.A	2	MC	Y
M2.1.B Solve problems that can be represented by systems of equations and inequalities.	A2.1.B	2	CP	Y
M2.1.C Solve problems that can be represented by quadratic functions, equations, and inequalities.	A1.1.D	2	CP	Y
M2.1.D Solve problems that can be represented by exponential functions and equations.	A1.1.E	2	MC,CP	Y
M2.1.E Solve problems involving combinations and permutations.	A1.1.F	2	MC,CP	Y

Key: *Format*= Multiple-Choice (MC), Completion (CP), Short-Answer (SA), or Not Assessed (NA)
C.C.= Cognitive Complexity (#) = Cognitive Complexity for italicized text
Ctxt=Contextual Situation I= Item dependent Y= Yes Context N= No Context

M2.2 Core Content: Quadratic functions, equations, and relationships

(Algebra)

Stimulus, Stem, and Prompt Rules

- Use Item Development Guidelines at the beginning of this document.
- Items assessing M2.2.F may expect students to give approximate solutions or numerically exact solutions. When an item does not specify, students may do either.
- Multiple-Choice items assessing M2.2.G will present complex roots in the form $x = \frac{-b \pm i\sqrt{d}}{2a}$ where b is an integer, d is a whole number, and a is a rational number.
- Items assessing M2.2.H may expect students to identify an equation that fits a set of bivariate data.

Content Expectations

Items may ask students to:	<i>Traditional Sequence</i>	<i>C.C.</i>	<i>Format</i>	<i>Ctxt</i>
M2.2.A Represent a quadratic function with a symbolic expression, as a graph, in a table, and with a description, and make connections among the representations.	A1.5.A	2	MC	I
M2.2.B Sketch the graph of a quadratic function, describe the effects that changes in the parameters have on the graph, and interpret the x-intercepts as solutions to a quadratic equation.	A1.5.B	1,2	MC	I
M2.2.C Translate between the standard form of a quadratic function, the vertex form, and the factored form; graph and interpret the meaning of each form.	A2.3.B	1,2	MC	N
M2.2.D Solve quadratic equations that can be factored as $(ax + b)(cx + d)$ where $a, b, c,$ and d are integers.	A1.5.C	1,2	MC,CP	N
M2.2.E Determine the number and nature of the roots of a quadratic function.	A2.3.B	1,2	MC	N
M2.2.F Solve quadratic equations that have real roots by completing the square and by using the quadratic formula.	A1.5.D	1,2	MC,CP	N
M2.2.G Solve quadratic equations and inequalities, including equations with complex roots.	A2.3.C	1,2	MC,CP	N
M2.2.H Determine if a bivariate data set can be better modeled with an exponential or a quadratic function and use the model to make predictions.	A2.6.E	2	MC	Y

Key: *Format*= Multiple-Choice (MC), Completion (CP), Short-Answer (SA), or Not Assessed (NA)
C.C.= Cognitive Complexity (#) = Cognitive Complexity for italicized text
Ctxt=Contextual Situation I= Item dependent Y= Yes Context N= No Context

M2.3 Core Content: Conjectures and proofs

(Algebra, Geometry/Measurement)

Stimulus, Stem, and Prompt Rules

- Use Item Development Guidelines at the beginning of this document.
- Items assessing M2.3 may require students to apply the theorems listed in the Theorems section at the end of this document.
- Items assessing M2.3 may include information about and require the use of exterior angles.
- Items assessing M2.3 may expect students to apply theorems to prove geometric statements, not to prove theorems.
- Items assessing M2.3 may expect students to complete a partial proof presented in two-column format, flow chart, or paragraph.
- Items assessing M2.3.C will not list the terms converse, inverse, and contrapositive as answer options.
- Items assessing M2.3.E may include similar triangles formed by a line segment parallel to one side of the triangle.
- Items assessing M2.3.E will not list the terms centroid, orthocenter, circumcenter and incenter as answer options.
- Items assessing M2.3.G may include figures that can be broken down into right triangles and other polygons.
- Items assessing M2.3.H may expect students to determine and/or label side lengths and/or angle measures of a right triangle.
- Multiple-Choice Items assessing M2.3.H may present answer choices in terms of sine, cosine, or tangent.
- Items assessing M2.3.L may expect students to determine the coordinates of a midpoint of a line segment, a vertex of a figure, or any other point described geometrically.
- Items assessing M2.3.L may expect students to represent the coordinates of a point in terms of the coordinates of other points.
- Items assessing M2.3.L and M2.3.M may include a blank grid; only the answer choice (Multiple-Choice Items) or the student answer (Completion Item) is scored.
- Items assessing M2.3.L may include a diagram on a coordinate plane.
- Items assessing M2.3.M may expect students to name triangles or quadrilaterals based on angle types, side lengths, and side relationships.

Performance Expectation M2.3 on the next page

Key: *Format*= Multiple-Choice (MC), Completion (CP), Short-Answer (SA), or Not Assessed (NA)
C.C.= Cognitive Complexity (#) = Cognitive Complexity for italicized text
Ctxt=Contextual Situation I= Item dependent Y= Yes Context N= No Context

Content Expectations

Items may ask students to:	Traditional Sequence	C.C.	Format	Ctxt
M2.3.A Use deductive reasoning to prove that a valid geometric statement is true.	G.1.C	2,3	*	*
M2.3.B Identify errors or gaps in a mathematical argument and develop counterexamples to refute invalid statements about geometric relationships.	G.1.E	2,3	MC	I
M2.3.C Write the converse, inverse, and contrapositive of a valid proposition and determine their validity.	G.1.D	2,3	MC,SA	Y
M2.3.D Distinguish between definitions and undefined geometric terms and explain the role of definitions, undefined terms, postulates (axioms), and theorems.	G.1.F	1	MC	N
M2.3.E Know, explain, and apply basic postulates and theorems about triangles and the special lines, line segments, and rays associated with a triangle.	G.3.A	2	MC,SA	N
M2.3.F Determine and prove triangle congruence and other properties of triangles.	G.3.B	2,3	MC,SA	N
M2.3.G Know, prove, and apply the Pythagorean Theorem and its converse.	G.3.D	1,2	MC,CP	I
M2.3.H Solve problems involving the basic trigonometric ratios of sine, cosine, and tangent.	G.3.E	1,2	MC,SA	I
M2.3.I Use the properties of special right triangles (30°–60°–90° and 45°–45°–90°) to solve problems.	G.3.C	1,2	MC,CP	I
M2.3.J Know, prove, and apply basic theorems about parallelograms.	G.3.F	2,3	MC,CP	I
M2.3.K Know, prove, and apply theorems about properties of quadrilaterals and other polygons.	G.3.G	2,3	MC,CP	I
M2.3.L Determine the coordinates of a point that is described geometrically.	G.4.B	1,2	MC,CP	I
M2.3.M Verify and apply properties of triangles and quadrilaterals in the coordinate plane.	G.4.C	1,2	MC,SA	I

* This performance expectation may be included in items assessing core process performance expectations or performance expectations that ask students to “know, prove, and apply” mathematical content.

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M2.4 Core Content: Probability

(Data/Statistics/Probability)

Stimulus, Stem, and Prompt Rules

- Use Item Development Guidelines at the beginning of this document.
- Items assessing M2.4.A may include independent and dependent events.

Content Expectations

Items may ask students to:	<i>Traditional Sequence</i>	<i>C.C.</i>	<i>Format</i>	<i>Ctxt</i>
M2.4.A Apply the fundamental counting principle and the ideas of order and replacement to calculate probabilities in situations arising from two-stage experiments (compound events).	A2.6.A	2	MC,CP	Y
M2.4.B Given a finite sample space consisting of equally likely outcomes and containing events A and B, determine whether A and B are independent or dependent, and find the conditional probability of A given B.	A2.6.B	2	MC,CP	Y
M2.4.C Compute permutations and combinations, and use the results to calculate probabilities.	A2.6.C	2	MC,CP	Y
M2.4.D Apply the binomial theorem to solve problems involving probability.	A2.6.D	2	MC,CP	Y

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M2.5 Additional Key Content

(Angles, Measurement)

Stimulus, Stem, and Prompt Rules

- Use Item Development Guidelines at the beginning of this document.
- Algebraic properties include the identity, zero, commutative, associative, and distributive properties.
- Items assessing M2.5 may expect students to write algebraic expressions in equivalent forms using algebraic properties and to perform the four arithmetic operations with polynomials.
- Items assessing M2.5.B may expect students to determine the level of precision that is appropriate for a given measurement tool.
- Items assessing M2.5.B will not ask students to estimate a calculation.
- Items assessing M2.5.C may include area measurements, volume measurements, and derived units of measure such as speed, density, flow rates, or population density.
- Items assessing M2.5.C that ask students to convert between systems will provide a conversion factor.

Content Expectations

Items may ask students to:	<i>Traditional Sequence</i>	<i>C.C.</i>	<i>Format</i>	<i>Ctxt</i>
M2.5.A Use algebraic properties to factor and combine like terms in polynomials.	A1.2.E	1	MC	N
M2.5.B Use different degrees of precision in measurement, explain the reason for using a certain degree of precision, and apply estimation strategies to obtain reasonable measurements with appropriate precision for a given purpose.	G.6.E	2	MC,SA	I
M2.5.C Solve problems involving measurement conversions within and between systems, including those involving derived units, and analyze solutions in terms of reasonableness of solutions and appropriate units.	G.6.F	2	MC,CP	I
M2.5.D Find the terms and partial sums of arithmetic and geometric series and the infinite sum for geometric series.	A2.7.B	2	MC,CP	N

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M2.6 Core Processes: Reasoning, problem solving, and communication

Stimulus, Stem, and Prompt Rules




- Use Item Development Guidelines at the beginning of this document.
- Problem solving items will include M2.6.A, M2.6.B, and M2.6.C.
- Items assessing M2.6.A-C may include figures that are not drawn to scale, include a statement "Picture is **not** drawn to scale."
- Items assessing M2.6.G may ask a student to support or contradict a given conclusion.
- Mathematics content for process items must be from performance expectations that are common to Mathematics 2 and Geometry.

Content Expectations

Items may ask students to:	<i>Traditional Sequence</i>	<i>C.C.</i>	<i>Format</i>	<i>Ctxt</i>
M2.6.A Analyze a problem situation and represent it mathematically.		3	MC,SA	Y
M2.6.B Select and apply strategies to solve problems.				
M2.6.C Evaluate a solution for reasonableness, verify its accuracy, and interpret the solution in the context of the original problem.				
M2.6.D <i>Generalize a solution strategy for a single problem to a class of related problems, and apply a strategy for a class of related problems to solve specific problems.</i>	G.7.D	(3,4)	NA	NA
M2.6.E Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.	G.7.E	2,3	SA	Y
M2.6.F <i>Summarize mathematical ideas with precision and efficiency for a given audience and purpose.</i>	G.7.F	(2,3)	NA	NA
M2.6.G Synthesize information to draw conclusions and evaluate the arguments and conclusions of others.	G.7.G	3	MC,SA	Y
M2.6.H <i>Use inductive reasoning to make conjectures, and use deductive reasoning to prove or disprove conjectures.</i>	G.7.H	(4)	NA	NA

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C.C.= Cognitive Complexity (#) = Cognitive Complexity for italicized text
Ctxt=Contextual Situation I= Item dependent Y= Yes Context N= No Context

Mathematics Symbols First Used in Exam Items

Strand	Name	Symbol	Grade
<i>Operations</i>	addition	+	3
	subtraction	-	3
	multiplication	×	3
		()	5
	division	•	6
		÷	3
		- (for ex. $\frac{6}{3}$)	4
	absolute value of a	$ a $	7
exponents	base exponent (for ex. 3^2)	8	
square root of a	\sqrt{a}	8	
<i>Algebra</i>	equal to	=	3
	less than	<	3
	greater than	>	3
	less than or equal to	≤	8
	greater than or equal to	≥	8
	not equal to	≠	8
	function	$f(x)$	EOC
	brackets	[]	EOC
<i>Geometric Sense</i>	right angle	 (in diagram)	3
	pi	π	6
	congruent	 (in diagram)	7
	line segment AB	\overline{AB}	7
	angle A	$\angle A, \angle BAC$	8
	triangle ABC	$\triangle ABC$	8
	perpendicular	\perp (in text)	8
	parallel	\parallel (in text)	8
		 (in diagram)	EOC
	similar	\sim	EOC
	measure of angle A	$m \angle A$	EOC
	congruent	\cong	EOC
	ray AB	\overrightarrow{AB}	EOC
	line AB	\overleftrightarrow{AB}	EOC
	arc AB	\overbrace{AB}	EOC

Mathematics Formula Sheets for End-of-Course Exams

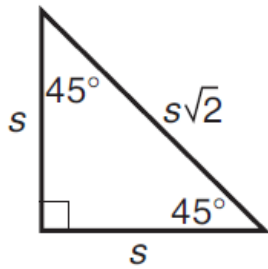
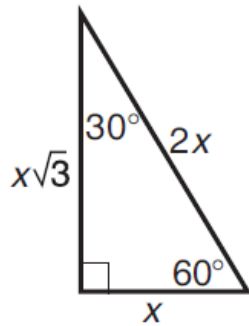
Use at least two decimal place values when approximating square roots or trigonometric ratios.

Description	Formula	Variables
Arc Length	$L = \frac{m\widehat{BC}}{360^\circ} \pi d$	L : Arc Length B, C : endpoints of arc d : diameter of the circle m : the measure of
Area of Sector	$A = \frac{m\widehat{BC}}{360} \pi r^2$	A : Area of Sector B, C : endpoints of intercepted arc r : radius of the circle m : the measure of
Cylinder	$SA = 2\pi r^2 + 2\pi rh$	SA : Surface Area r : radius of the base h : height
	$V = \pi r^2 h$	V : Volume r : radius of the base h : height
Cone	$SA = \pi r^2 + \pi rl$	SA : Surface Area r : radius of the base l : slant height
	$V = \frac{1}{3} Bh$ or $V = \frac{1}{3} \pi r^2 h$	V : Volume r : radius of the base h : height B : area of the base
Prism	$V = Bh$	V : Volume B : area of the base H : height
	$SA = 2B + Ph$ or $SA = 2B + L$	SA : Surface Area B : area of the base P : Perimeter of the base h : height L : lateral surface area
Pyramid	$V = \frac{1}{3} Bh$	V : Volume B : area of the base h : height
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	x : solution a, b, c : coefficients
Sphere	$V = \frac{4}{3} \pi r^3$	V : Volume r : radius
	$SA = 4\pi r^2$	SA : Surface Area r : radius

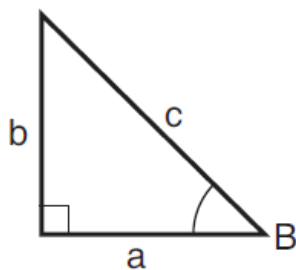
Mathematics Formula Sheets for End-of-Course Exams

Use at least two decimal place values when approximating square roots or trigonometric ratios.

Special Right Triangles



Trigonometric Ratios

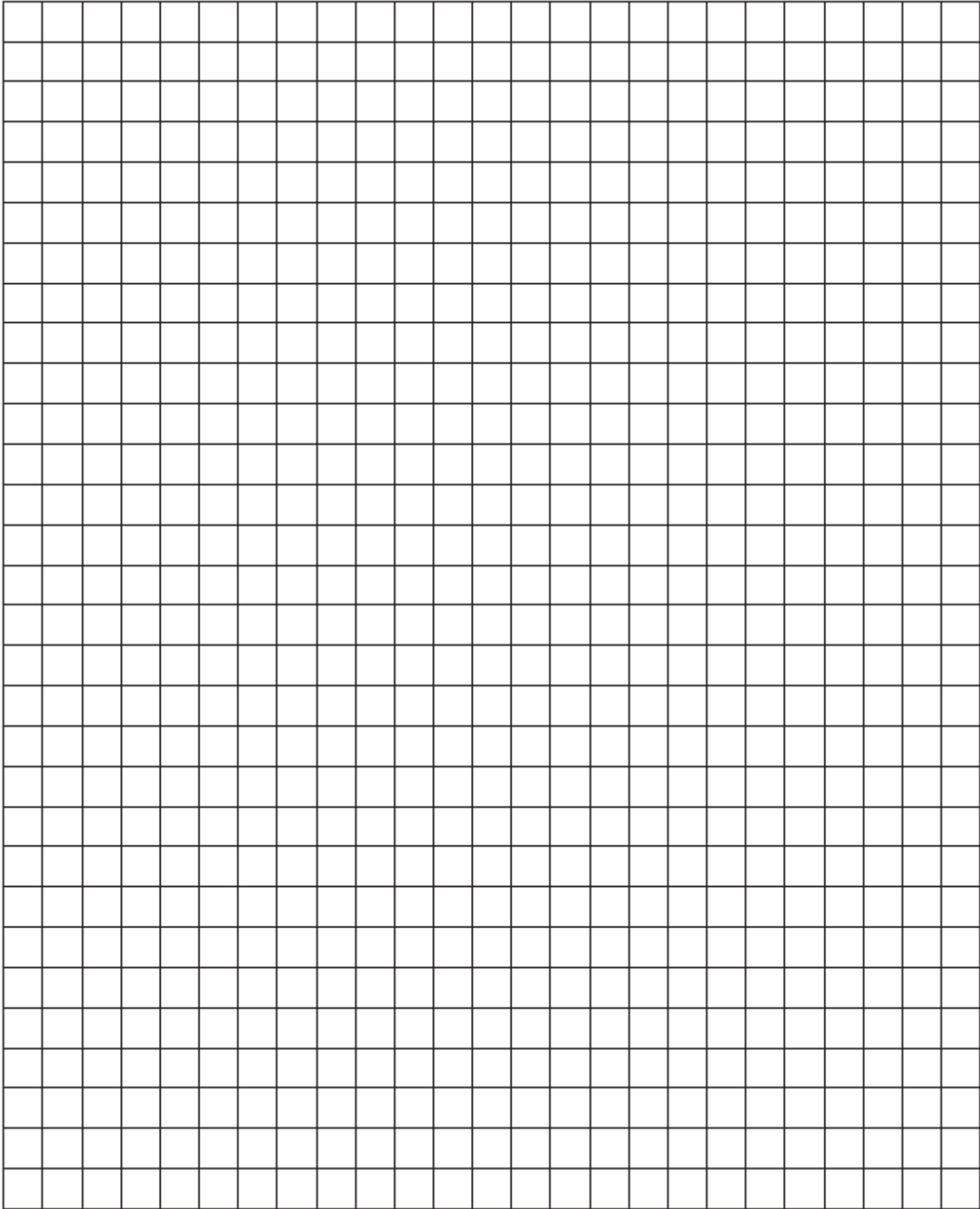


$$\sin B = \frac{b}{c}$$

$$\cos B = \frac{a}{c}$$

$$\tan B = \frac{b}{a}$$

Mathematics End-of-Course Graph Paper



Measurement Vocabulary

Attributes, Units, Abbreviations, and Grade Level First-Used in Assessment Items

The levels in parentheses refer to the grades at which students should develop an oral understanding of the terms according to the K-2 Standards.

Attribute	Unit	Grade
Length (K)	inch (in.)	3 (2)
	foot (ft)	3 (2)
	yard (yd)	3 (2)
	mile (mi)	4
	millimeter (mm)	4
	centimeter (cm)	3 (2)
	meter (m)	3 (2)
	kilometer (km)	4
Capacity (K)	cup (c)	3
	pint (pt)	3
	quart (qt)	3
	gallon (gal)	3
	milliliter (mL)	3
	liter (L)	3
	kiloliter (kL)	4
Weight (K)	ounce (oz)	3
	pound (lb)	3
	ton (t)	7
Mass	gram (g)	3
	kilogram (kg)	4

Attribute	Unit	Grade
Time	second (s)	4
	minute (min)	4 (2)
	hour (h)	4 (2)
	day (d)	4 (2)
	week (wk)	4 (2)
	month (mo)	4 (2)
	year (y)	4 (2)
Temperature	Degree Fahrenheit (°F)	3
	Degree Celsius (°C)	3
Angle	Degrees (°)	5

Conversions

- In grades 4-6, students are expected to convert within a measurement system but not between measurement systems. For example, 3 feet = 1 yard is a conversion within the U.S. customary system, but 1 yard \approx 0.91 meter is a conversion between U.S. Customary and metric systems.
- In grade 7 and Geometry/Integrated Mathematics 2 EOCs students may be asked to convert between systems and a conversion factor will be provided.
- Most dictionaries list conversion factors.

This list of definitions, postulates and theorems describes geometric concepts students are expected to know, understand and apply. Students are not expected to know or memorize the exact wording of the items in this list. This list will not be provided during the test.

Definitions, Postulates and Theorems

M2.3.E: Know, explain, and apply basic postulates and theorems about triangles and the special lines, line segments, and rays associated with a triangle.

1. A triangle is isosceles if and only if the base angles are congruent.
2. The medians of a triangle are concurrent at the centroid.
3. The lines containing the altitudes of a triangle are concurrent at the orthocenter.
4. The lines containing the perpendicular bisector of a triangle are concurrent at the circumcenter. Their common point is equidistant from the three vertices of the triangle.
5. The lines containing the angle bisectors of a triangle are concurrent at the incenter. Their common point is equidistant from the three sides of the triangle.
6. The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote angles.
7. The sum of the length of any two sides of a triangle is greater than the length of the third side.
8. In a triangle, the longest side is opposite the largest angle.
9. In a triangle, the largest angle is opposite the longest side.

M2.3.F: Determine and prove triangle congruence and other properties of triangles.

1. If three sides of one triangle are congruent to three sides of another triangle, then the two triangles are congruent. (Side – Side – Side Congruence)
2. If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent. (Side – Angle – Side Congruence)
3. If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent. (Angle – Side – Angle Congruence)
4. If two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of another triangle, then the two triangles are congruent. (Angle – Angle – Side Congruence)

M2.3.J: Know, prove, and apply basic theorems about parallelograms.

1. A quadrilateral is a parallelogram if and only if its opposite sides are congruent.
2. A quadrilateral is a parallelogram if and only if its opposite angles are congruent.
3. A quadrilateral is a parallelogram if and only if its consecutive interior angles are supplementary.
4. A quadrilateral is a parallelogram if and only if its diagonals bisect each other.
5. A parallelogram is a rectangle if and only if its diagonals are congruent.

6. A parallelogram is a rhombus if and only if its diagonals are perpendicular.
7. A parallelogram is a rhombus if and only if its diagonals bisect pairs of opposite angles.

M2.3.K: Know, prove, and apply theorems about properties of quadrilaterals and other polygons.

1. If a figure is a trapezoid, then consecutive angles between a pair of parallel lines are supplementary.
2. If a figure is a kite, then the diagonals are perpendicular.
3. The sum of one set of the exterior angles of a polygon is 360° .
4. The sum of the interior angles of a polygon is $((n-2)180)^\circ$ where n is the number of sides of the polygon.