Washington State K–8 Mathematics Standards
April 2008

Processes Strand

The development of mathematical thinking, reasoning, and problem solving is the essence of school mathematics. Students need a balanced foundation of understanding mathematical ideas and knowing mathematical skills and procedures. But this knowledge in itself is inadequate if students cannot use what they know to think through problems that allow them to apply and extend what they know. The process strand includes similar expectations from year to year—especially within K–2, 3–5, and 6–8 grade bands—but it gradually calls for increasing sophistication of student thinking and the application of the thinking and reasoning skills to content specific to each grade level.

This is one of six strand documents that accompany the Washington State K–8 Mathematics Standards, tracking the development of important mathematical ideas and skills across grades K–8. Where content of an expectation may address more than one strand, that expectation may appear in more than one strand document.

Kindergarten

K.5. Core Processes: *Reasoning, problem solving, and communication*

Students begin to build the understanding that doing mathematics involves solving problems and discussing how they solved them. Problems at this level emphasize counting and activities that lead to emerging ideas about addition and subtraction. Students begin to develop their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?"

Performance Expectations

Students are expected to:

K.5.A Identify the question(s) asked in a problem.

K.5.B Identify the given information that can be used to solve a problem.

K.5.C Recognize when additional information is required to solve a problem.

K.5.D Select from a variety of problem-solving strategies and use one or more strategies to solve a problem.

K.5.E Answer the question(s) asked in a problem.

K.5.F Describe how a problem was solved.

K.5.G Determine whether a solution to a problem is reasonable.

Explanatory Comments and Examples

Descriptions of solution processes and explanations can include numbers, words (including mathematical language), pictures, or physical objects. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.

Examples:

- Grandma went to visit her two grandchildren and discovered that the gloves they were each wearing had holes in every finger, even the thumbs. She will fix their gloves. How many glove fingers (including thumbs) need to be fixed?
- Students are given drinking straws or coffee stirrers cut to a variety of different lengths: 6″, 5″, 4″, 3″, and 2″. They are to figure out which sets of three lengths, when joined at their ends, will form triangles and which sets of three will not.

Grade 1

1.6. Core Processes: *Reasoning, problem solving, and communication*

Students further develop the concept that doing mathematics involves solving problems and discussing what they did to solve them. Problems in first grade emphasize addition, subtraction, and solidifying number concepts, and sometimes include precursors to multiplication. Students continue to develop their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?"; "Why did you do that?"; and "How do you know that?" Students begin to build their mathematical vocabulary as they use correct mathematical language appropriate to first grade.

Performance Expectations

Students are expected to:

1.6.A Identify the question(s) asked in a problem.

1.6.B Identify the given information that can be used to solve a problem.

1.6.C Recognize when additional information is required to solve a problem.

1.6.D Select from a variety of problem-solving strategies and use one or more strategies to solve a problem.

1.6.E Answer the question(s) asked in a problem.

1.6.F Identify the answer(s) to the question(s) in a problem.

1.6.G Describe how a problem was solved.

1.6.H Determine whether a solution to a problem is reasonable.

Explanatory Comments and Examples

Descriptions of solution processes and explanations can include numbers, words (including mathematical language), pictures, or physical objects. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.

Examples:

- Think about how many feet a person has. How many feet does a cat have? How many feet does a snail have? How about a fish or a snake?
  - There are ten feet living in my house. Who could be living in my house?
  - Come up with a variety of ways you can have a total of ten feet living in your house. Use pictures, words, or numbers to show how you got your answer.
- You are in charge of setting up a dining room with exactly twenty places for people to sit. You can use any number and combination of different-shaped tables. A hexagon-shaped table seats six people. A triangle-shaped table seats three people. A square-shaped table seats four people.
  - Draw a picture showing which tables and how many of each you could set up so that twenty people have a place to sit. Is there more than one way to do this? How many ways can you find?

Grade 2

2.5. Core Processes: *Reasoning, problem solving, and communication*

Students further develop the concept that doing mathematics involves solving problems and talking about what they did to solve those problems. Second-grade problems emphasize addition and subtraction with increasingly large numbers, measurement, and early concepts of multiplication and division. Students communicate their mathematical thinking and make increasingly more convincing mathematical arguments. Students participate in mathematical discussions involving questions like "How did you get that?"; "Why did you use that strategy?"; and "Why is that true?" Students continue to build their mathematical vocabulary as they use correct mathematical language appropriate to grade two when discussing and refining solutions to problems.

Performance Expectations

Students are expected to:

2.5.A Identify the question(s) asked in a problem and any other questions that need to be answered in order to solve the problem.

2.5.B Identify the given information that can be used to solve a problem.

2.5.C Recognize when additional information is required to solve a problem.

2.5.D Select from a variety of problem-solving strategies and use one or more strategies to solve a problem.

2.5.E Identify the answer(s) to the question(s) in a problem.

2.5.F Describe how a problem was solved.

2.5.G Determine whether a solution to a problem is reasonable.

Explanatory Comments and Examples

Descriptions of solution processes and explanations can include numbers, words (including mathematical language), pictures, or physical objects. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.

Examples:

- A bag full of jellybeans is on the table. There are 10 black jellybeans in the bag. There are twice as many red jellybeans as black jellybeans. There are 2 fewer red jellybeans than yellow jellybeans. There are half as many pink jellybeans as yellow jellybeans. How many jellybeans are in the bag? Explain your answer.
- Suzy, Ben, and Pedro have found 1 quarter, 1 dime, and 4 pennies under the sofa. Their mother has lots of change in her purse, so they could trade any of these coins for other coins adding up to the same value. She says they can keep the money if they can tell her what coins they need so the money can be shared equally among them. How can they do this?

Grade 3

3.6. Core Processes: *Reasoning, problem solving, and communication*

Students in grade three solve problems that extend their understanding of core mathematical concepts—such as geometric figures, fraction concepts, and multiplication and division of whole numbers—as they make strategic decisions that bring them to reasonable solutions. Students use pictures, symbols, or mathematical language to explain the reasoning behind their decisions and solutions. They further develop their problem-solving skills by making generalizations about the processes used and applying these generalizations to similar problem situations. These critical reasoning, problem-solving, and communication skills represent the kind of mathematical thinking that equips students to use the mathematics they know to solve a growing range of useful and important problems and to make decisions based on quantitative information.

Performance Expectations

Students are expected to:

3.6.A Determine the question(s) to be answered given a problem situation.

3.6.B Identify information that is given in a problem and decide whether it is necessary or unnecessary to the solution of the problem.

3.6.C Identify missing information that is needed to solve a problem.

3.6.D Determine whether a problem to be solved is similar to previously solved problems, and identify possible strategies for solving the problem.

3.6.E Select and use one or more appropriate strategies to solve a problem.

3.6.F Represent a problem situation using words, numbers, pictures, physical objects, or symbols.

3.6.G Explain why a specific problem-solving strategy or procedure was used to determine a solution.

3.6.H Analyze and evaluate whether a solution is reasonable, is mathematically correct, and answers the question.

3.6.I Summarize mathematical information, draw conclusions, and explain reasoning.

3.6.J Make and test conjectures based on data (or information) collected from explorations and experiments.

Explanatory Comments and Examples

Descriptions of solution processes and explanations can include numbers, words (including mathematical language), pictures, physical objects, or equations. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.

Examples:

- Whitney wants to put a fence around the perimeter of her square garden. She plans to include a gate that is 3 ft wide. The length of one side of the garden is 19 ft. The fencing comes in two sizes: rolls that are 18 ft long and 24 ft long. Which rolls and how many of each should Whitney buy in order to have the least amount of leftover fencing? Justify your answer.
- A soccer team is selling water bottles with soccer balls painted on them to raise money for new equipment. The team bought 10 boxes of water bottles. Each box cost $27 and had 9 bottles. At what price should the team sell each bottle in order to make $180 profit to pay for new soccer balls? Justify your answer.

Grade 4

4.5. Core Processes: *Reasoning, problem solving, and communication*

Students in grade four solve problems that extend their understanding of core mathematical concepts—such as multiplication of multi-digit numbers, area, probability, and the relationships between fractions and decimals—as they make strategic decisions that bring them to reasonable solutions. Students use pictures, symbols, or mathematical language to explain the reasoning behind their decisions and solutions. They further develop their problem-solving skills by making generalizations about the processes used and applying these generalizations to similar problem situations. These critical reasoning, problem-solving, and communication skills represent the kind of mathematical thinking that equips students to use the mathematics they know to solve a growing range of useful and important problems and to make decisions based on quantitative information.

Performance Expectations

Students are expected to:

4.5.A Determine the question(s) to be answered given a problem situation.

4.5.B Identify information that is given in a problem and decide whether it is essential or extraneous to the solution of the problem.

4.5.C Identify missing information that is needed to solve a problem.

4.5.D Determine whether a problem to be solved is similar to previously solved problems, and identify possible strategies for solving the problem.

4.5.E Select and use one or more appropriate strategies to solve a problem and explain why that strategy was chosen.

4.5.F Represent a problem situation using words, numbers, pictures, physical objects, or symbols.

4.5.G Explain why a specific problem-solving strategy or procedure was used to determine a solution.

4.5.H Analyze and evaluate whether a solution is reasonable, is mathematically correct, and answers the question.

4.5.I Summarize mathematical information, draw conclusions, and explain reasoning.

4.5.J Make and test conjectures based on data (or information) collected from explorations and experiments.

Explanatory Comments and Examples

Descriptions of solution processes and explanations can include numbers, words (including mathematical language), pictures, physical objects, or equations. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.

Examples:

- Jake's family adopted a small dog, Toto. They have a rectangular dog pen that is 10 feet by 20 feet. Toto needs only half that area, so Jake plans to make the pen smaller by cutting each dimension in half. Jake's mother asked him to rethink his plan or Toto won't have the right amount of space.
  - Whose reasoning is correct—Jake's or his mother's? Why?
  - According to Jake's plan, what fractional part of the old pen will be the area of the new pen? Give the answer in simplest form.
  - Make a new plan so that the area of the new pen is half the area of the old pen.
- The city is paying for a new deck around the community pool. The rectangular pool measures 50 meters by 25 meters. The deck, which will measure 5 meters wide, will surround the pool like a picture frame. If the cost of the deck is $25 for each square meter, what will be the total cost for the new deck? Explain your solution.

Grade 5

5.6. Core Processes: *Reasoning, problem solving, and communication*

Students in grade five solve problems that extend their understanding of core mathematical concepts—such as division of multi-digit numbers, perimeter, area, addition and subtraction of fractions and decimals, and use of variables in expressions and equations—as they make strategic decisions leading to reasonable solutions. Students use pictures, symbols, or mathematical language to explain the reasoning behind their decisions and solutions. They further develop their problem-solving skills by making generalizations about the processes used and applying these generalizations to similar problem situations. These critical reasoning, problem-solving, and communication skills represent the kind of mathematical thinking that equips students to use the mathematics they know to solve a growing range of useful and important problems and to make decisions based on quantitative information.

Performance Expectations

Students are expected to:

5.6.A Determine the question(s) to be answered given a problem situation.

5.6.B Identify information that is given in a problem and decide whether it is essential or extraneous to the solution of the problem.

5.6.C Determine whether additional information is needed to solve the problem.

5.6.D Determine whether a problem to be solved is similar to previously solved problems, and identify possible strategies for solving the problem.

5.6.E Select and use one or more appropriate strategies to solve a problem, and explain the choice of strategy.

5.6.F Represent a problem situation using words, numbers, pictures, physical objects, or symbols.

5.6.G Explain why a specific problem-solving strategy or procedure was used to determine a solution.

5.6.H Analyze and evaluate whether a solution is reasonable, is mathematically correct, and answers the question.

5.6.I Summarize mathematical information, draw conclusions, and explain reasoning.

Explanatory Comments and Examples

Descriptions of solution processes and explanations can include numbers, words (including mathematical language), pictures, physical objects, or equations. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.

Examples:

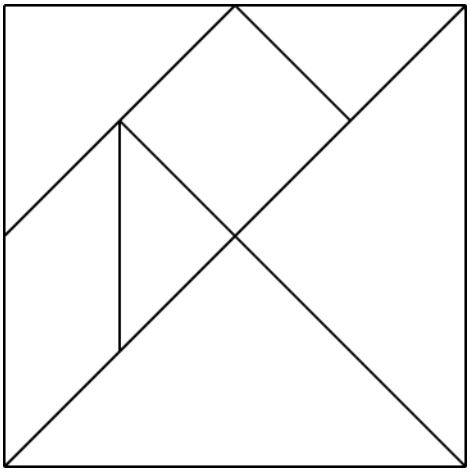
- La Casa Restaurant uses rectangular tables. One table seats 6 people, with 1 person at each end and 2 people on each long side. However, 2 tables pushed together, short end to short end, seat only 10 people. Three tables pushed together end-to-end seat only 14 people. Write a rule that describes how many can sit at n tables pushed together end-to-end. The restaurant's long banquet hall has tables pushed together in a long row to seat 70. How many tables were pushed together to seat this many people? How do you know?
- The small square in the tangram figure below is $\frac{1}{8}$ the area of the large square.
  - For each of the 7 tangram pieces that make up the large square, tell what fractional part of the large square that piece represents. How do you know?

Performance Expectations

Students are expected to:

5.6.J Make and test conjectures based on data (or information) collected from explorations and experiments.

Explanatory Comments and Examples



Grade 6

6.6. Core Processes: *Reasoning, problem solving, and communication*

Students refine their reasoning and problem-solving skills as they move more fully into the symbolic world of algebra and higher-level mathematics. They move easily among representations—numbers, words, pictures, or symbols—to understand and communicate mathematical ideas, to make generalizations, to draw logical conclusions, and to verify the reasonableness of solutions to problems. In grade six, students solve problems that involve fractions and decimals as well as rates and ratios in preparation for studying proportional relationships and algebraic reasoning in grade seven.

Performance Expectations

Students are expected to:

6.6.A Analyze a problem situation to determine the question(s) to be answered.

6.6.B Identify relevant, missing, and extraneous information related to the solution to a problem.

6.6.C Analyze and compare mathematical strategies for solving problems, and select and use one or more strategies to solve a problem.

6.6.D Represent a problem situation, describe the process used to solve the problem, and verify the reasonableness of the solution.

6.6.E Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.

6.6.F Apply a previously used problem-solving strategy in a new context.

6.6.G Extract and organize mathematical information from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.

6.6.H Make and test conjectures based on data (or information) collected from explorations and experiments.

Explanatory Comments and Examples

Descriptions of solution processes and explanations can include numbers, words (including mathematical language), pictures, physical objects, or equations. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.

Examples:

- As part of her exercise routine, Carmen jogs twice around the perimeter of a square park that measures $\frac{5}{8}$ mile on each side. On Monday, she started at one corner of the park and jogged $\frac{2}{3}$ of the way around in 17 minutes before stopping at a small pond in the park to feed some ducks. How far had Carmen run when she reached the pond? What percent of her planned total distance had Carmen completed when she stopped to feed the ducks? If it took Carmen 17 minutes to jog to the point where she stopped, assuming that she continued running in the same direction at the same pace and did not stop again, how long would it have taken her to get back to her starting point? Explain your answers.

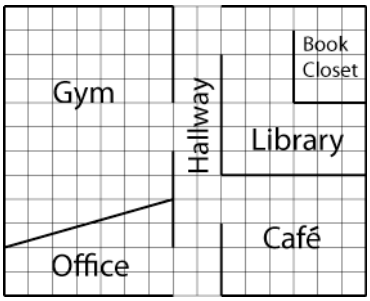
Performance Expectations

Students are expected to:

Explanatory Comments and Examples

- At Springhill Elementary School's annual fair, Vanessa is playing a game called "Find the Key." A key is randomly placed somewhere in one of the rooms shown on the map below. (The key cannot be placed in the hallway.)

  To win the game, Vanessa must correctly guess the room where the key is placed. Use what you know about the sizes of the rooms to determine the probability that the key is placed in the gym, the office, the café, the book closet, or the library. Write each probability as a simplified fraction, a decimal, and a percent. Which room should Vanessa select in order to have the best chance of winning? Justify the solution.



Grade 7

7.6. Core Processes: *Reasoning, problem solving, and communication*

Students refine their reasoning and problem-solving skills as they move more fully into the symbolic world of algebra and higher-level mathematics. They move easily among representations—numbers, words, pictures, or symbols—to understand and communicate mathematical ideas, to make generalizations, to draw logical conclusions, and to verify the reasonableness of solutions to problems. In grade seven, students solve problems that involve positive and negative numbers and often involve proportional relationships. As students solve these types of problems, they build a strong foundation for the study of linear functions that will come in grade eight.

Performance Expectations

Students are expected to:

7.6.A Analyze a problem situation to determine the question(s) to be answered.

7.6.B Identify relevant, missing, and extraneous information related to the solution to a problem.

7.6.C Analyze and compare mathematical strategies for solving problems, and select and use one or more strategies to solve a problem.

7.6.D Represent a problem situation, describe the process used to solve the problem, and verify the reasonableness of the solution.

7.6.E Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.

7.6.F Apply a previously used problem-solving strategy in a new context.

7.6.G Extract and organize mathematical information from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.

7.6.H Make and test conjectures based on data (or information) collected from explorations and experiments.

Explanatory Comments and Examples

Descriptions of solution processes and explanations can include numbers, words (including mathematical language), pictures, physical objects, or equations. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.

Examples:

- When working on a report for class, Catrina read that a person over the age of 30 can lose approximately 0.06 centimeters of height per year. Catrina's 80-year-old grandfather is 5 feet 7 inches tall. Assuming her grandfather's height has decreased at this rate, about how tall was he at age 30? Catrina's cousin, Richard, is 30 years old and is 6 feet 3 inches tall. Assuming his height also decreases approximately 0.06 centimeters per year after the age of 30, about how tall will you expect him to be at age 55? (Remember that 1 inch $\approx$ 2.54 centimeters.) Justify your solution.
- If one man takes 1.5 hours to dig a 5-ft $\times$ 5-ft $\times$ 3-ft hole, how long will it take three men working at the same pace to dig a 10-ft $\times$ 12-ft $\times$ 3-ft hole? Explain your solution.

Grade 8

8.5. Core Processes: *Reasoning, problem solving, and communication*

Students refine their reasoning and problem-solving skills as they move more fully into the symbolic world of algebra and higher level mathematics. They move easily among representations—numbers, words, pictures, or symbols—to understand and communicate mathematical ideas, to make generalizations, to draw logical conclusions, and to verify the reasonableness of solutions to problems. In grade eight, students solve problems that involve proportional relationships and linear relationships, including applications found in many contexts outside of school. These problems dealing with proportionality continue to be important in many applied contexts, and they lead directly to the study of algebra. Students also begin to deal with informal proofs for theorems that will be proven more formally in high school.

Performance Expectations

Students are expected to:

8.5.A Analyze a problem situation to determine the question(s) to be answered.

8.5.B Identify relevant, missing, and extraneous information related to the solution to a problem.

8.5.C Analyze and compare mathematical strategies for solving problems, and select and use one or more strategies to solve a problem.

8.5.D Represent a problem situation, describe the process used to solve the problem, and verify the reasonableness of the solution.

8.5.E Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.

8.5.F Apply a previously used problem-solving strategy in a new context.

8.5.G Extract and organize mathematical information from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.

8.5.H Make and test conjectures based on data (or information) collected from explorations and experiments.

Explanatory Comments and Examples

Descriptions of solution processes and explanations can include numbers, words (including mathematical language), pictures, or equations. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.

Examples:

- The dimensions of a room are 12 feet by 15 feet by 10 feet. What is the furthest distance between any two points in the room? Explain your solution.
- Miranda's phone service contract ends this month. She is looking for ways to save money and is considering changing phone companies. Her current cell phone carrier, X-Cell, calculates the monthly bill using the equation c = $15.00 + $0.07m, where c represents the total monthly cost and m represents the number of minutes of talk time during a monthly billing cycle. Another company, Prism Cell, offers 300 free minutes of talk time each month for a base fee of $30.00 with an additional $0.15 for every minute over 300 minutes. Miranda's last five phone bills were $34.95, $36.70, $37.82, $62.18, and $36.28. Using the data from the last five months, help Miranda decide whether she should switch companies. Justify your answer.