

Washington State K–8 Mathematics Standards April 2008

Geometry/Measurement Strand

In kindergarten through grade 2, students become familiar with locations in space and describe and compare two- and three-dimensional figures. They compare objects and begin to learn what it means to measure something, beginning with length. Students in grades 3–5 continue to learn about special characteristics of geometric figures, including symmetry and congruence. They compute the perimeters and areas of figures, and they develop volume concepts by computing volumes of rectangular prisms. In grades 6–8, students connect what they know about measurement skills to geometric ideas, solving problems involving π , the Pythagorean Theorem, and similarity. By the end of high school, students know and can prove properties of two- and three-dimensional geometric figures, and they solve increasingly complex problems involving geometric figures and their measurements.

This is one of six strand documents that accompany the Washington State K–8 Mathematics Standards, tracking the development of important mathematical ideas and skills across grades K–8. Where content of an expectation may address more than one strand, that expectation may appear in more than one strand document.

Kindergarten

K.3. Core Content: *Objects and their locations*

Performance Expectations

Students are expected to:

- K.3.A Identify, name, and describe circles, triangles, rectangles, squares (as special rectangles), cubes, and spheres.
- K.3.B Sort shapes using a sorting rule and explain the sorting rule.
- K.3.C Describe the location of one object relative to another object using words such as *in, out, over, under, above, below, between, next to, behind,* and *in front of.*

Explanatory Comments and Examples

Students should be encouraged to talk about the characteristics (e.g., round, four-cornered) of the various shapes and to identify these shapes in a variety of contexts regardless of their location, size, and orientation. Having students identify these shapes on the playground, in the classroom, and on clothing develops their ability to generalize the characteristics of each shape.

Students could sort shapes according to attributes such as the shape, size, or the number of sides, and explain the sorting rule. Given a selection of shapes, students may be asked to sort them into two piles and then describe the sorting rule. After sorting, a student could say, "I put all the round ones here and all the others there."

Examples:

- Put this pencil under the paper.
- I am between José and Katy.

Kindergarten

K.4. Additional Key Content

Performance Expectations

Students are expected to:

- K.4.A Make direct comparisons using measurable attributes such as length, weight, and capacity.

Explanatory Comments and Examples

Students should use language such as longer than, shorter than, taller than, heavier than, lighter than, holds more than, or holds less than.

Grade 1

1.3. Core Content: *Geometric attributes*

Performance Expectations

Students are expected to:

- 1.3.A Compare and sort a variety of two- and three-dimensional figures according to their geometric attributes.

Explanatory Comments and Examples

The student may sort a collection of two-dimensional figures into those that have a particular attribute (e.g., those that have straight sides) and those that do not.

Performance Expectations

Students are expected to:

1.3.B Identify and name two-dimensional figures, including those in real-world contexts, regardless of size or orientation.

1.3.C Combine known shapes to create shapes and divide known shapes into other shapes.

Explanatory Comments and Examples

Figures should include circles, triangles, rectangles, squares (as special rectangles), rhombi, hexagons, and trapezoids.

Contextual examples could include classroom clocks, flags, desktops, wall or ceiling tiles, etc. Triangles should appear in many positions and orientations and should not all be equilateral or isosceles.

Students could be asked to trace objects or use a drawing program to show different ways that a rectangle can be divided into three triangles. They can also use pattern blocks or plastic shapes to make new shapes. The teacher can give students cutouts of shapes and ask students to combine them to make a particular shape.

Example:

- What shapes can be made from a rectangle and a triangle? Draw a picture to show your answers.

Grade 1

1.4. Core Content: Concepts of measurement

Performance Expectations

Students are expected to:

1.4.A Recognize that objects used to measure an attribute (length, weight, capacity) must be consistent in size.

1.4.B Use a variety of non-standard units to measure length.

1.4.C Compare lengths using the transitive property.

1.4.D Use non-standard units to compare objects according to their capacities or weights.

Explanatory Comments and Examples

Marbles can be suitable objects for young children to use to measure weight, provided that all the marbles are the same weight. Paper clips are appropriate for measuring length as long as the paper clips are all the same length.

Use craft sticks, toothpicks, coffee stirrers, etc., to measure length.

Example:

- If Jon is taller than Jacob, and Jacob is taller than Luisa, then Jon is taller than Luisa.

Examples can include using filled paper cups to measure capacity or a balance with marbles or cubes to measure weight.

Performance Expectations

Students are expected to:

1.4.E Describe the connection between the size of the measurement unit and the number of units needed to measure something.

1.4.F Name the days of the week and the months of the year, and use a calendar to determine a day or month.

Grade 2

2.2. Core Content: Addition and subtraction

Performance Expectations

Students are expected to:

2.2.H Name each standard U.S. coin, write its value using the \$ sign and the ¢ sign, and name combinations of other coins with the same total value.

2.2.I Determine the value of a collection of coins totaling less than \$1.00.

Explanatory Comments and Examples

Examples:

- It takes more toothpicks than craft sticks to measure the width of my desk. The longer the unit, the fewer I need.
- It takes fewer marbles than cubes to balance my object. The lighter the unit, the more I need.
- It takes more little medicine cups filled with water than larger paper cups filled with water to fill my jar. The less my unit holds, the more I need.

Examples:

- Name the days of the week in order.
- Name the months of the year in order.
- How many days until your birthday?
- What month comes next?
- What day was it yesterday?

Explanatory Comments and Examples

Students should be expected to express, for example, the value of a quarter as twenty-five cents, \$0.25, and 25¢, and they should be able to give other combinations of coins whose value is 25¢. This is a precursor to decimal notation.

Grade 2

2.3. Core Content: Measurement

Performance Expectations

Students are expected to:

- 2.3.A Identify objects that represent or approximate standard units and use them to measure length.
- 2.3.B Estimate length using metric and U.S. customary units.
- 2.3.C Measure length to the nearest whole unit in both metric and U.S. customary units.
- 2.3.D Describe the relative size among minutes, hours, days, weeks, months, and years.
- 2.3.E Use both analog and digital clocks to tell time to the minute.

Explanatory Comments and Examples

At this level, students no longer rely on non-standard units. Students find and use approximations for standard length units, like a paper clip whose length is about an inch, or the width of a particular student's thumbnail that might be about a centimeter. They might also use commonly available classroom objects like inch tiles or centimeter cubes.

Students could make observations such as, "The ceiling of the classroom is about 8 feet high."

Standard tools may include rulers, yardsticks, meter sticks, or centimeter/inch measuring tapes. Students should measure some objects that are longer than the measurement tool being used.

Students should be able to describe relative sizes using statements like, "Since a minute is less than an hour, there are more minutes than hours in one day."

Grade 2

2.4. Additional Key Content

Performance Expectations

Students are expected to:

- 2.4.A Solve problems involving properties of two- and three-dimensional figures.

Explanatory Comments and Examples

A critical component in the development of students' spatial and geometric understanding is the ability to solve problems involving the properties of figures. At the primary level, students must move from judging plane and space shapes by their appearance as whole shapes to focusing on the relationship of the sides, angles, or faces. At the same time, students must learn the language important for describing shapes according to their essential characteristics. Later, they will describe properties of shapes in more formal ways as they progress in geometry.

Performance Expectations

Students are expected to:

Explanatory Comments and Examples

Examples:

- How many different ways can you fill the outline of the figure with pattern blocks? What is the greatest number of blocks you can use? The least number? Can you fill the outline with every whole number of blocks between the least number of blocks and the greatest number of blocks?
- Build a figure or design out of five blocks. Describe it clearly enough so that someone else could build it without seeing it. Blocks may represent two-dimensional figures (i.e., pattern blocks) or three-dimensional figures (i.e., wooden geometric solids).

Grade 3

3.4. Core Content: Geometry

Performance Expectations

Students are expected to:

- 3.4.A Identify and sketch parallel, intersecting, and perpendicular lines and line segments.
- 3.4.B Identify and sketch right angles.
- 3.4.C Identify and describe special types of quadrilaterals.
- 3.4.D Measure and calculate perimeters of quadrilaterals.
- 3.4.E Solve single- and multi-step word problems involving perimeters of quadrilaterals and verify the solutions.

Explanatory Comments and Examples

Special types of quadrilaterals include squares, rectangles, parallelograms, rhombi, trapezoids and kites.

Example:

- Sketch a parallelogram with two sides 9 cm long and two sides 6 cm long. What is the perimeter of the parallelogram?

Examples:

- Julie and Jacob have recently created two rectangular vegetable gardens in their backyard. One garden measures 6 ft by 8 ft, and the other garden measures 10 ft by 5 ft. They decide to place a small fence around the outside of each garden to prevent their dog from getting into their new vegetables. How many feet of fencing should Julie and Jacob buy to fence both gardens?

Grade 3

3.5. Additional Key Content

Performance Expectations

Students are expected to:

- 3.5.B Measure temperature in degrees Fahrenheit and degrees Celsius using a thermometer.
- 3.5.C Estimate, measure, and compare weight and mass using appropriate-sized U.S. customary and metric units.
- 3.5.D Estimate, measure, and compare capacity using appropriate-sized U.S. customary and metric units.

Explanatory Comments and Examples

The scale on a thermometer is essentially a vertical number line. Students may informally deal with negative numbers in this context, although negative numbers are not formally introduced until grade six.

Measure temperature to the nearest degree.

Grade 4

4.3. Core Content: Concept of area

Performance Expectations

Students are expected to:

- 4.3.A Determine congruence of two-dimensional figures.
- 4.3.B Determine the approximate area of a figure using square units.

Explanatory Comments and Examples

At this grade level, students determine congruence primarily by making direct comparisons (i.e., tracing or cutting). They may also use informal notions of transformations described as flips, turns, and slides. Both the language and the concepts of transformations are more formally developed in grade eight.

Examples:

- Draw a rectangle 3.5 cm by 6 cm on centimeter grid paper. About how many squares fit inside the rectangle?
- Cover a footprint with square tiles or outline it on grid paper. About how many squares fit inside the footprint?

Performance Expectations

Students are expected to:

4.3.C Determine the perimeter and area of a rectangle using formulas, and explain why the formulas work.

4.3.D Determine the areas of figures that can be broken down into rectangles.

4.3.E Demonstrate that rectangles with the same area can have different perimeters, and that rectangles with the same perimeter can have different areas.

4.3.F Solve single- and multi-step word problems involving perimeters and areas of rectangles and verify the solutions.

Explanatory Comments and Examples

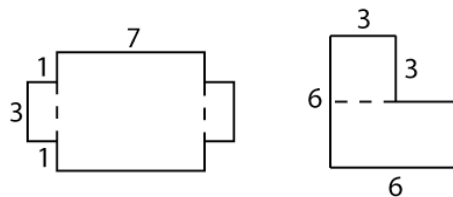
This is an opportunity to connect area to the concept of multiplication, a useful model for multiplication that extends into algebra. Students should also work with squares as special rectangles.

Example:

- Outline on grid paper a rectangle that is 4 units long and 3 units wide. Without counting the squares, how can you determine the area? Other than measuring, how could you use a shortcut to find the perimeter of the rectangle?

Example:

- Find the area of each figure:



Example:

- Draw different rectangles, each with an area of 24 square units, and compare their perimeters. What patterns do you notice in the data? Record your observations.

The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations.

Problems include those involving U.S. customary and metric units, including square units.

Grade 4

4.4. Additional Key Content

Performance Expectations

Students are expected to:

- 4.4.B Solve single- and multi-step problems involving familiar unit conversions, including time, within either the U.S. customary or metric system.
- 4.4.C Estimate and determine elapsed time using a calendar, a digital clock, and an analog clock.

Grade 5

5.3. Core Content: Triangles and quadrilaterals

Performance Expectations

Students are expected to:

- 5.3.A Classify quadrilaterals.
- 5.3.B Identify, sketch, and measure acute, right, and obtuse angles.
- 5.3.C Identify, describe, and classify triangles by angle measure and number of congruent sides.

Explanatory Comments and Examples

Examples:

- Jill bought 3 meters of ribbon and cut it into pieces 25 centimeters long. How many 25-centimeter pieces of ribbon did she have?
- How many quarts of lemonade are needed to make 25 one-cup servings?

Explanatory Comments and Examples

Students sort a set of quadrilaterals into their various types, including parallelograms, kites, squares, rhombi, trapezoids, and rectangles, noting that a square can also be classified as a rectangle, parallelogram, and rhombus.

Example:

- Use a protractor to measure the following angles and label each as acute, right, or obtuse.



Students classify triangles by their angle size using the terms *acute*, *right*, or *obtuse*.

Students classify triangles by the length of their sides using the terms *scalene*, *isosceles*, or *equilateral*.

Performance Expectations

Students are expected to:

5.3.D Determine the formula for the area of a parallelogram by relating it to the area of a rectangle.

5.3.E Determine the formula for the area of a triangle by relating it to the area of a parallelogram.

5.3.F Determine the perimeters and areas of triangles and parallelograms.

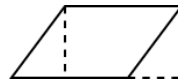
5.3.G Draw quadrilaterals and triangles from given information about sides and angles.

5.3.H Determine the number and location of lines of symmetry in triangles and quadrilaterals.

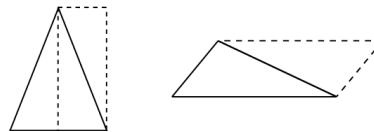
5.3.I Solve single- and multi-step word problems about the perimeters and areas of quadrilaterals and triangles and verify the solutions.

Explanatory Comments and Examples

Students relate the area of a parallelogram to the area of a rectangle, as shown below.



Students relate the area of a triangle to the area of a parallelogram, as shown below.



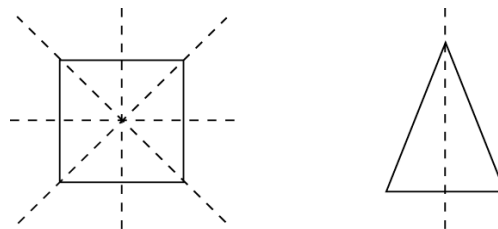
Students may be given figures showing some side measures or may be expected to measure sides of figures. If students are not given side measures, but instead are asked to make their own measurements, it is important to discuss the approximate nature of any measurement.

Examples:

- Draw a triangle with one right angle and no congruent sides.
- Draw a rhombus that is not a square.
- Draw a right scalene triangle.

Example:

- Draw and count all the lines of symmetry in the square and isosceles triangle below. (Lines of symmetry are shown as dotted lines.)



The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations.

Grade 6

6.3. Core Content: Ratios, rates, and percents

Performance Expectations

Students are expected to:

- 6.3.E Identify the ratio of the circumference to the diameter of a circle as the constant π , and recognize $\frac{22}{7}$ and 3.14 as common approximations of π .

Explanatory Comments and Examples

Example:

- Measure the diameter and circumference of several circular objects. Divide each circumference by its diameter. What do you notice about the results?

Grade 6

6.4. Core Content: Two- and three-dimensional figures

Performance Expectations

Students are expected to:

- 6.4.A Determine the circumference and area of circles.

- 6.4.B Determine the perimeter and area of a composite figure that can be divided into triangles, rectangles, and parts of circles.

Explanatory Comments and Examples

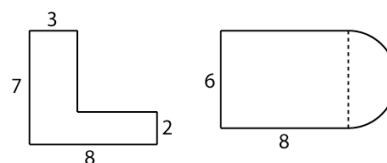
Examples:

- Determine the area of a circle with a diameter of 12 inches.
- Determine the circumference of a circle with a radius of 32 centimeters.

Although students have worked with various quadrilaterals in the past, this expectation includes other quadrilaterals such as trapezoids or irregular quadrilaterals, as well as any other composite figure that can be divided into figures for which students have calculated areas before.

Example:

- Determine the area and perimeter of each of the following figures, assuming that the dimensions on the figures are in feet. The curved portion of the second figure is a semi-circle.



Performance Expectations

Students are expected to:

6.4.C Solve single- and multi-step word problems involving the relationships among radius, diameter, circumference, and area of circles, and verify the solutions.

6.4.D Recognize and draw two-dimensional representations of three-dimensional figures.

6.4.E Determine the surface area and volume of rectangular prisms using appropriate formulas and explain why the formulas work.

6.4.F Determine the surface area of a pyramid.

6.4.G Describe and sort polyhedra by their attributes: parallel faces, types of faces, number of faces, edges, and vertices.

Explanatory Comments and Examples

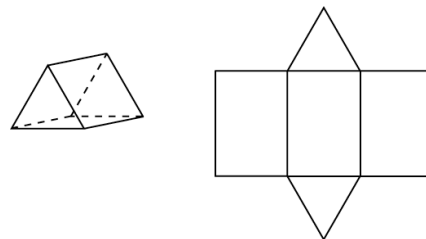
The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations.

Example:

- Captain Jenkins determined that the distance around a circular island is 44 miles. What is the distance from the shore to the buried treasure in the center of the island? What is the area of the island?

The net of a rectangular prism consists of six rectangles that can then be folded to make the prism. The net of a cylinder consists of two circles and a rectangle.

Example:

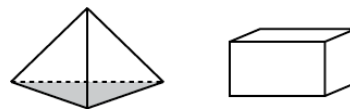


Students may determine surface area by calculating the area of the faces and adding the results.

Prisms and pyramids are the focus at this level.

Examples:

- How many pairs of parallel faces does each polyhedron have? Explain your answer.



- What type of polyhedron has two parallel triangular faces and three non-parallel rectangular faces?

Grade 7

7.2. Core Content: Proportionality and similarity

Performance Expectations

Students are expected to:

7.2.C Describe proportional relationships in similar figures and solve problems involving similar figures.

7.2.D Make scale drawings and solve problems related to scale.

7.2.I Solve single- and multi-step problems involving conversions within or between measurement systems and verify the solutions.

Explanatory Comments and Examples

Students should recognize the constant ratios in similar figures and be able to describe the role of a scale factor in situations involving similar figures. They should be able to connect this work with more general notions of proportionality.

Example:

- The length of the shadow of a tree is 68 feet at the same time that the length of the shadow of a 6-foot vertical pole is 8 feet. What is the height of the tree?

Example:

- On an 80:1 scale drawing of the floor plan of a house, the dimensions of the living room are

$1\frac{7}{8}$ '' $2\frac{1}{2}$ '' . What is the actual area of the living room in square feet?

The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations.

Students should be given the conversion factor when converting between measurement systems.

Examples:

- The lot that Dana is buying for her new one-story house is 35 yards by 50 yards. Dana's house plans show that her house will cover 1,600 square feet of land. What percent of Dana's lot will not be covered by the house? Explain your work.

Performance Expectations

Students are expected to:

Grade 7

7.3. Core Content: Surface Area and Volume

Performance Expectations

Students are expected to:

- 7.3.A Determine the surface area and volume of cylinders using the appropriate formulas and explain why the formulas work.
- 7.3.B Determine the volume of pyramids and cones using formulas.
- 7.3.C Describe the effect that a change in scale factor on one attribute of a two- or three-dimensional figure has on other attributes of the figure, such as the side or edge length, perimeter, area, surface area, or volume of a geometric figure.
- 7.3.D Solve single- and multi-step word problems involving surface area or volume and verify the solutions.

Explanatory Comments and Examples

- Joe was planning a business trip to Canada, so he went to the bank to exchange \$200 U.S. dollars for Canadian dollars (at a rate of \$1.02 CDN per \$1 US). On the way home from the bank, Joe's boss called to say that the destination of the trip had changed to Mexico City. Joe went back to the bank to exchange his Canadian dollars for Mexican pesos (at a rate of 10.8 pesos per \$1 CDN). How many Mexican pesos did Joe get?

Explanatory Comments and Examples

Explanations might include the use of models such as physical objects or drawings.

A net can be used to illustrate the formula for finding the surface area of a cylinder.

Examples:

- A cube has a side length of 2 cm. If each side length is tripled, what happens to the surface area? What happens to the volume?
- What happens to the area of a circle if the diameter is decreased by a factor of 3?

The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations.

Examples:

- Alexis needs to paint the four exterior walls of a large rectangular barn. The length of the barn is 80 feet, the width is 50 feet, and the height is 30 feet. The paint costs \$28 per gallon, and each gallon covers 420 square feet. How much will it cost Alexis to paint the barn? Explain your work..

Performance Expectations

Students are expected to:

Explanatory Comments and Examples

- Tyesha has decided to build a solid concrete pyramid on her empty lot. The base will be a square that is forty feet by forty feet and the height will be thirty feet. The concrete that she will use to construct the pyramid costs \$70 per cubic yard. How much will the concrete for the pyramid cost Tyesha? Justify your answer.

Grade 8

8.2. Core Content: Properties of geometric figures

Performance Expectations

Students are expected to:

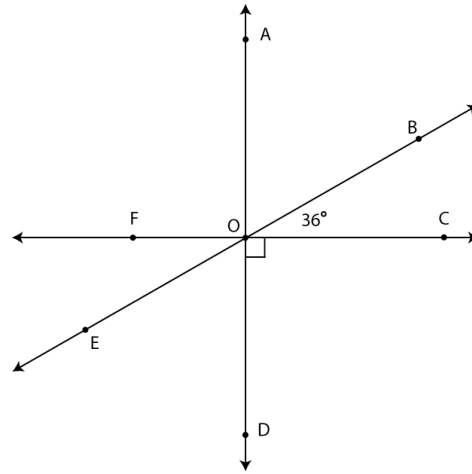
- 8.2.A Identify pairs of angles as complementary, supplementary, adjacent, or vertical, and use these relationships to determine missing angle measures.

- 8.2.B Determine missing angle measures using the relationships among the angles formed by parallel lines and transversals.

Explanatory Comments and Examples

Example:

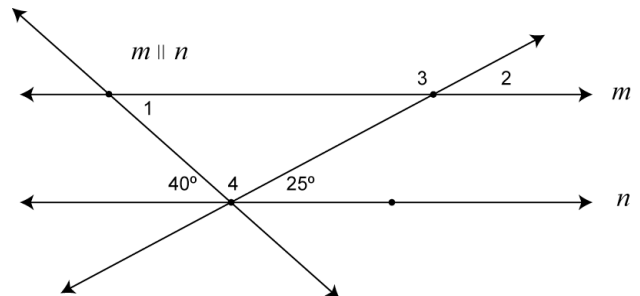
- Determine the measures of $\angle BOA$, $\angle EOD$, $\angle FOB$, and $\angle FOE$ and explain how you found each measure. As part of your explanation, identify pairs of angles as complementary, supplementary, or vertical.



Example:

- Determine the measures of the indicated angles.

$\angle 1$: _____ $\angle 2$: _____ $\angle 3$: _____ $\angle 4$: _____



Performance Expectations

Students are expected to:

8.2.C Demonstrate that the sum of the angle measures in a triangle is 180 degrees, and apply this fact to determine the sum of the angle measures of polygons and to determine unknown angle measures.

8.2.D Represent and explain the effect of one or more translations, rotations, reflections, or dilations (centered at the origin) of a geometric figure on the coordinate plane.

8.2.F Demonstrate the Pythagorean Theorem and its converse and apply them to solve problems.

8.2.G Apply the Pythagorean Theorem to determine the distance between two points on the coordinate plane.

Explanatory Comments and Examples

Examples:

- Determine the measure of each interior angle in a regular pentagon.
- In a certain triangle, the measure of one angle is four times the measure of the smallest angle, and the measure of the remaining angle is the sum of the measures of the other two angles. Determine the measure of each angle.

Example:

- Consider a trapezoid with vertices (1,2), (1,6), (6,4), and (6,2). The trapezoid is reflected across the x-axis and then translated four units to the left. Graph the image of the trapezoid after these two transformations and give the coordinates of the new vertices.

One possible demonstration is to start with a right triangle, use each of the three triangle sides to form the side of a square, and draw the remaining three sides of each of the three squares. The areas of the three squares represent the Pythagorean relationship.

Examples:

- Is a triangle with side lengths 5 cm, 12 cm, and 13 cm a right triangle? Why or why not?
- Determine the length of the diagonal of a rectangle that is 7 ft by 10 ft.

Example:

- Determine the distance between the points (–2, 3) and (4, 7).