Guidelines for accelerating students into high school mathematics in grade 8

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Introduction

Washington's mathematics standards define the mathematics that should be taught and learned in kindergarten through grade 8 and in high school courses following grade 8, beginning with Algebra 1 or Mathematics 1. Some schools and school districts have chosen to accelerate some or all students into the high school series of courses beginning in grade 8, rather than grade 9. These guidelines are intended to provide recommendations for how to best maintain the continuity and strength of a student's mathematics preparation when students are accelerated at this level.

Background

Historically in the United States, the organization of the mathematics curriculum included considerable review of elementary mathematics in the middle school grades. The intent of this *spiral curriculum* was to have students build on what they know as they expand their knowledge. However, in reality, the typical American mathematics curriculum has tended to repeat and review each year's learning without much expansion. This has led to a rushed and often superficial treatment of mathematics, with teachers pushed to 'cover' material and with students rarely having adequate opportunity to learn well the topics to which they are exposed. Thus, middle school mathematics has often been a time of revisiting elementary mathematics for most students. At the same time, students who had learned the intended mathematics in grades K–6 have sometimes been accelerated into high school mathematics (Algebra 1 or Integrated Math 1) in grade 8, rather than the traditional grade 9.

In recent years, statistics have shown an alarming and unacceptable imbalance in the demographics of students enrolled in both advanced and remedial mathematics in high school. Disproportionate numbers of economically disadvantaged students and students of color are often represented in remedial high school mathematics courses or in a lower track. Likewise, disproportionate numbers of white, Asian, and economically advantaged students tend to be represented in advanced high school mathematics courses, especially Advanced Placement and International Baccalaureate programs that require five years of high school mathematics. In order to provide all students the opportunity for such advanced study, and, consequently, the further opportunities provided by completing these programs, many schools have implemented the practice of accelerating students into high school mathematics programs beginning in grade 8. The results are mixed in terms of whether these students are likely to pursue mathematics through these advanced options, but the opportunity to accelerate certainly opens the door for this possibility.

Recent comparisons with high-performing nations around the world have shown that the United States has tended to address topics related to algebra somewhat later than other countries. This observation, compounded with a growing push from business leaders and policy makers for a stronger mathematics program for all American students that prepares them for global competitiveness in the future, has resulted in a call for more students, or even all students, to study algebra in grade 8.

The situation many communities have faced in recent years, then, includes a repetitive math curriculum, inadequate access to advanced mathematics, and international comparisons that show other countries dealing with algebra and other high school content earlier than the United States. At the same time communities are choosing to accelerate more (or all) students into high school mathematics at grade 8 (or sooner, in some cases), developers of mathematics curriculum materials have taken on these same issues throughout the K–12 mathematics program, especially at the elementary and middle school levels. Washington's mathematics standards reflect this more current thinking, and, thus, decisions about whether to accelerate students and which students should be accelerated should be made with the full awareness of the rapidly changing scene of K–8 mathematics in the state of Washington.

Washington's K–8 Mathematics Standards

The 2008 Washington K–8 mathematics standards demonstrate a strong commitment to the expectation of more mathematics for students at every grade level. Algebraic thinking, a long-time strength of Washington's standards, is included at every grade level. Furthermore, the standards include considerable content traditionally addressed in high school algebra, especially in the grades 7 and 8 mathematics standards.

The 2008 middle school standards represent a strong set of expectations for all students. They include a considerable amount of new content, well beyond a standard review of previously learned mathematics. Students completing the K–8 Washington mathematics standards will have an unprecedented foundation for success in high school mathematics, and they will have addressed many topics formerly reserved for high school algebra or its integrated equivalent (including solving equations and using linear functions to model situations and solve a range of applied problems).

The issue of accelerating students into high school mathematics at grade 8 must therefore be reviewed from a different perspective than in years past. The decision to accelerate a student or group of students carries some advantages, as noted above, but also bring about new cautions in light of advances in how we define K–12 mathematics, as reflected in the Washington standards.

Compressing Middle School Mathematics

The attached guidelines show how mathematics content from grades 7 and 8 could be organized into a single compressed course in grade 7. Educators and parents should note that it will be challenging to provide the depth and focus built into the grades 7 and 8 standards in this compressed format; schools will want to ensure that adequate instructional time, teacher professional development, and student support programs are in place, as described below.

In compressing the grades 7 and 8 expectations, topics have been reorganized in a single sequence that may not always reflect the same sequence as either of the original grades. Out of necessity, there are more expectations within each Core Content area in this compressed one-year program than in the original two-year organization, and there are a total of five Core Content areas, rather than three or four. Also note that while there is no section called *Additional Key Content*, there is a section labeled *Content that may be revisited in grade 8 or 9, or in Algebra 1, Geometry, Math 1, or Math 2*. This section includes content that teachers may consider treating differently as they work with teachers at other grade levels. This content is by no means unimportant, but teachers may wish to work collaboratively with teachers of these other courses in determining how to best prepare students for success at these levels. Note that this content may still be eligible to be tested, depending on test specifications when the expectations are adopted.

Following the list of compressed expectations combining grades 7 and 8 into a single year, the last section of this document includes some suggestions for sliding some topics from the compressed grade 7 and 8 expectations into grade 6. Modifying the grade 6 mathematics standards in this way is not necessarily recommended because it can track students too early into an accelerated path and, thus, can eliminate some students who might otherwise be successful. However, in those few cases where a school has decided to place all students into Algebra 1 or Mathematics 1 in grade 8, these suggestions may be helpful.

Issues and Considerations in Accelerating Students into High School Mathematics

The discussion below may guide educators, parents, and administrators in making decisions about whether students should be accelerated, which students should be accelerated, and issues related to placement and future success in mathematics. Because these decisions will have significant impact on students for many years, they should be made carefully, especially in light of new, more rigorous K–8 mathematics standards.

Should Students Enroll in Algebra 1/Math 1 in Grade 8?

Students interested in enrolling in Advanced Placement calculus or other five-year high school mathematics options can benefit from beginning their high school sequence of mathematics courses in grade 8. This document includes a suggested reorganization of the Washington mathematics standards for grades 7 and 8 into a single year as preparation for acceleration of students into Algebra 1 or Mathematics 1 in grade 8, as well as additional considerations and guidelines for such a reorganization. In particular, parents, schools, and school districts should consider several issues when making such a decision.

- **Ensure that all students receive the strong foundation in K–8 mathematics that will prepare them for success in high school mathematics.**

  Washington's 2008 K–8 mathematics standards raise the bar in terms of what is expected of all students. Important content related to understanding proportional relationships and building data analysis skills prepares students for success in high school mathematics. Much of what has traditionally been considered algebra content is part of the K–8 standards, especially in grades 7 and 8. If students are to begin their study of high school mathematics in grade 8, schools should take steps to ensure that these students have adequate opportunities to learn the full breadth and, more important, depth of the K–8 standards. If students are rushed through the middle school standards in the interest of acceleration, they may develop only a superficial knowledge of important mathematical content, including the critical topic of proportional relationships. Without a strong background on proportionality and other topics, students are likely to struggle as they move into the high school program. The guidelines in this document provide a way of organizing a compression of middle school mathematics to accommodate this acceleration, but the actual learning that takes place depends on a strong support structure for both students and teachers, such as those described below.

- **Do not rush decisions to accelerate a student into high school mathematics in grade 8.**

  All students need to have options for their future for as long as possible, at least through grade 5. Making decisions to track students into an accelerated or remedial track should not be made early in a student's elementary program, or some potentially successful students might be denied access to higher level options. Consequently, these guidelines primarily address a one-year compression of the standards for grades 7 and 8, allowing the acceleration decision to be made as late as possible. If a school decides to accelerate all students into algebra in grade 8, these guidelines suggest topics that might be moved to grade 6 in order to more reasonably balance the mathematics content load in grades 6 and 7.

- **Make placement decisions based on a set of clear, consistent, transparent, objective criteria.**

  It is tempting to accelerate students based on factors such as teacher recommendations or parental requests. However, decisions made in this way can tend to overlook students who may have the academic background to succeed, but who may not fit the model of a traditionally ‘good’ student or who may not have an active parental advocate. There is no single picture of what kind of student will succeed in high school mathematics. Elusive assumptions of aptitude, achievement, attitude, and behavior all find counterexamples in the faces of students who have succeeded when some thought they would not. Further, a single placement test may not fully reflect what a student knows and is able to do. Thus, it is advisable to establish a set of criteria and/or placement rubric that might include consideration of such items as test scores or a portfolio of student work. For this type of decision, it is generally not appropriate to heavily weight such factors as attendance, attitude, or maturity, as these may not be related to a student’s potential for success in high school mathematics. Overall, students should have a strong and balanced foundation in understanding mathematical concepts, carrying out mathematical procedures, and applying mathematics to solve a range of problems.

  Nationally, some districts are finding that they also can use their state mathematics test scores or other quasi-objective measures to inform such decisions. Regardless of the particular criteria used for accelerated mathematics placement, when these criteria are standardized across schools, previous inequities in placement among racial groups can be nearly eliminated without diminishing the quality of mathematics learning.

- **Provide teacher and student support and a school structure conducive to optimal mathematics learning with depth and lasting understanding and proficiency.**

  When students are accelerated into high school mathematics in grade 8, teachers of both grades 7 and 8 are handed a significant responsibility. Teaching a compressed grade 7 and 8 program in one year calls for sophisticated teaching expertise and deep mathematical knowledge if students are to gain the depth of understanding and proficiency that will serve them well throughout their extended study of mathematics in high school. Furthermore, teachers of high school mathematics in grade 8 need the same preparation and knowledge as other teachers of high school mathematics. The challenges of teaching this kind of “fast-track” mathematics, even to very capable students, demand a lot from teachers. Schools and districts have a responsibility to provide extensive and ongoing professional development focused on mathematics content and appropriate ways to help students learn that content. Inadequate class time typically found in middle schools often compounds the problems many middle school teachers already face in adequately preparing students on an accelerated track. It is unreasonable to expect teachers to teach in depth or students to learn twice as much mathematics in grade 7 with class periods of 40–45 minutes or to learn high school Algebra 1 or Math 1 in class periods this short. Even block periods of 90 minutes, when offered for only half as many days, fall short of what is needed for deep and lasting learning of mathematics. In recent policy statements, the National Council of Teachers of Mathematics has called for the equivalent of 60 minutes a day in mathematics, and the College Board has called for at least 55 minutes a day for students in grades 6 through 12 in order for them to learn the increased mathematics called for today at the national level and reflected in the Washington standards.

- **Provide challenging and relevant options in grades 11 and 12 to students who begin their high school mathematics study in grade 8.**

  By the end of grade 10, students who begin their high school mathematics study in grade 8 could complete their expected three-year sequence of mathematics through Algebra 1, Geometry, and Algebra 2 or through (integrated) Mathematics 1/2/3. Most recommendations call for students to study mathematics every year they are in school through grade 12. So, it becomes critical that schools offer students at least two years of challenging and appropriate mathematics courses that follow these first three years. Certainly pre-calculus and calculus represent one path. But students not headed toward a mathematically intensive field after high school need different choices. If a student does not have reasonable options for five solid years of mathematics in high school, the decision to accelerate that student into the high school sequence in grade 8 should be reconsidered, or else the system should commit to providing such options by the time the student reaches grade 11. Without these options in place, students may grow bored or frustrated in inappropriate courses or may lose some of what they learn before they have a chance to apply it after high school.

- **Examine the demographics of students being accelerated compared to students in remedial classes and students in the school's and district's overall population.**

  The demographic breakdown of students in accelerated mathematics should reflect the demographics of the school and district. If students of color or economically disadvantaged students are not proportionally represented in the group of students being accelerated, the district should closely examine the mathematics program in terms of placement criteria and whether students and teachers are adequately supported to help all students learn rigorous mathematics.

- **Issues of acceleration or compression should be considered in light of attention to the entire K–12 mathematics program.**

  Accelerating students into high school mathematics in grade 8 is but one aspect of the K–12 mathematics program. In order to accomplish its intended goal (more students taking advanced mathematics and science courses in high school), significant attention should be given to making sure both the K–8 and the high school mathematics program are accomplishing intended goals. This requires ongoing professional development, high quality instructional materials, a well-aligned system of testing (both formative and summative), and a support structure to ensure success for all students at every grade level. Without this systemic attention to the mathematics program, students may not only fall short of benefiting from the acceleration, they may in fact be worse off than if they had not been accelerated. On the other hand, as part of an overall effort to improve K–12 mathematics, allowing students the opportunity to study advanced mathematics can be an important component to increased success for all students in rigorous, relevant, appropriate mathematics.

  The most critical element for a student's future success is not when the student begins high school mathematics, but rather the quality of instruction the student receives every year of school. A student who learns mathematics well each year will undoubtedly succeed the following year. More important, that student will graduate high school able to use mathematics in every facet of life and ready for a range of opportunities either in college or in the work place.

Suggested Reading

Zal Usiskin, “Why Elementary Algebra Can, Should, and Must Be an Eighth-Grade Course for Average Students,” *Mathematics Teacher*, September 1987.

Cathy Seeley, “Pushing Algebra Down,” President’s Message, National Council of Teachers of Mathematics *News Bulletin*, March 2005.

Compressing Expectations from Grades 7 and 8 into Grade 7 for a One-Year Program Leading to Algebra 1 or Mathematics 1 in Grade 8

Grade 7/8 compression

7/8.1. Core Content: *Rational numbers and number theory*

Performance Expectations

Students are expected to:

7.1.A Compare and order rational numbers using the number line, lists, and the symbols <, >, or =.

7.1.B Represent addition, subtraction, multiplication, and division of positive and negative integers visually and numerically.

7.1.C Fluently and accurately add, subtract, multiply, and divide rational numbers.

7.2.A Mentally add, subtract, multiply, and divide simple fractions, decimals, and percents.

7.1.D Define and determine the absolute value of a number.

7.1.G Solve single- and multi-step word problems involving rational numbers and verify the solutions.

7.5.B Write the prime factorization of whole numbers greater than 1, using exponents when appropriate.

8.4.C Evaluate numerical expressions involving non-negative integer exponents using the laws of exponents and the order of operations.

8.4.A Represent numbers in scientific notation, and translate numbers written in scientific notation into standard form.

8.4.B Solve problems involving operations with numbers in scientific notation and verify solutions.

8.2.E Quickly recall the square roots of the perfect squares from 1 through 225 and estimate the square roots of other positive numbers. [See the comment in the right column regarding the connection to the Pythagorean Theorem.]

8.4.D Identify rational and irrational numbers. [See the comment in the right column regarding the connection to the Pythagorean Theorem.]

Grade 7/8 compression

7/8.2. Core Content: *Linear equations and functions* *(Algebra)*

Performance Expectations

Students are expected to:

7.1.E (includes 7.1.E and 8.1.A) Solve two-step linear equations in one variable.

7.1.F Write an equation that corresponds to a given problem situation, and describe a problem situation that corresponds to a given equation.

8.1.C Represent a linear function with a verbal description, table, graph, or symbolic expression, and make connections among these representations.

8.1.G Determine and justify whether a given verbal description, table, graph, or symbolic expression represents a linear relationship.

8.1.F Solve single- and multi-step word problems involving linear functions and verify the solutions.

Grade 7/8 compression

7/8.3. Core Content: *Proportionality and similarity*

Performance Expectations

Students are expected to:

7.2.B Solve single- and multi-step problems involving proportional relationships and verify the solutions.

7.2.C Describe proportional relationships in similar figures and solve problems involving similar figures.

7.2.D Make scale drawings and solve problems related to scale.

7.2.E Represent proportional relationships using graphs, tables, and equations, and make connections among the representations.

7.2.F (partial) Determine the slope of a line corresponding to the graph of a proportional relationship. [Note the second part of this expectation is listed under 7/8.6. Additional Key Content.]

7.2.G Determine the unit rate in a proportional relationship and relate it to the slope of the associated line.

7.2.H Determine whether or not a relationship is proportional and explain your reasoning.

7.2.I Solve single- and multi-step problems involving conversions within or between measurement systems and verify the solutions.

7.3.C Describe the effect that a change in scale factor on one attribute of a two- or three-dimensional figure has on other attributes of the figure, such as the side or edge length, perimeter, area, surface area, or volume of a geometric figure.

Grade 7/8 compression

7/8.4. Core Content: *Properties and relationships in geometric figures*

Performance Expectations

Students are expected to:

7.3.A Determine the surface area and volume of cylinders using the appropriate formulas and explain why the formulas work.

7.3.B Determine the volume of pyramids and cones using formulas.

7.3.D Solve single- and multi-step word problems involving surface area or volume and verify the solutions.

8.2.A Identify pairs of angles as complementary, supplementary, adjacent, or vertical, and use these relationships to determine missing angle measures.

8.2.B Determine missing angle measures using the relationships among the angles formed by parallel lines and transversals.

8.2.C Demonstrate that the sum of the angle measures in a triangle is 180 degrees, and apply this fact to determine the sum of the angle measures of polygons and to determine unknown angle measures.

7.5.A Graph ordered pairs of rational numbers and determine the coordinates of a given point in the coordinate plane.

8.2.D Represent and explain the effect of one or more translations, rotations, reflections, or dilations (centered at the origin) of a geometric figure on the coordinate plane.

8.2.F Demonstrate the Pythagorean Theorem and its converse and apply them to solve problems.

8.2.G Apply the Pythagorean Theorem to determine the distance between two points on the coordinate plane.

Grade 7/8 Compression

7/8.5. Core Content: *Probability and data*

Performance Expectations

Students are expected to:

7.4.C (includes 7.4.C and 8.3.A) Use measures of center (median, mean, and mode) and variability (maximum, minimum, and range) to describe a data set or summarize and compare data sets, and evaluate the suitability and limitations of using each measure for different situations.

7.4.D Construct and interpret histograms, stem-and-leaf plots, and circle graphs.

7.4.E Evaluate different displays of the same data for effectiveness and bias, and explain reasoning.

8.3.B Select, construct, and analyze data displays, including box-and-whisker plots, to compare two sets of data.

8.3.C Create a scatterplot for a two-variable data set, and, when appropriate, sketch and use a trend line to make predictions.

8.3.D Describe different methods of selecting statistical samples and analyze the strengths and weaknesses of each method.

8.3.E Determine whether conclusions of statistical studies reported in the media are reasonable.

7.4.A Represent the sample space of probability experiments in multiple ways, including tree diagrams and organized lists.

7.4.B Determine the theoretical probability of a particular event and use theoretical probability to predict experimental outcomes.

8.3.F Determine probabilities for mutually exclusive, dependent, and independent events for small sample spaces.

8.3.G Solve single- and multi-step problems using counting techniques and Venn diagrams and verify the solutions.

Grade 7/8 compression

7/8.6. Content that may be *revisited in grade 8 or 9, or in Algebra 1, Geometry, Math 1, or Math 2*

Performance Expectations

Students are expected to:

8.1.B Solve one- and two-step linear inequalities and graph the solutions on the number line.

8.1.D Determine the slope and *y*-intercept of a linear function described by a symbolic expression, table, or graph.

8.1.E Interpret the slope and *y*-intercept of the graph of a linear function representing a contextual situation.

7.2.F (partial) Relate slope to similar triangles. [Note that the other part of 7.2.F is included in 7/8.3 on proportionality.]

Grade 7/8 Compression

7/8.7. Core Processes: *Reasoning, problem solving, and communication*

Performance Expectations

Students are expected to:

7.6.A Analyze a problem situation to determine the question(s) to be answered.

7.6.B Identify relevant, missing, and extraneous information related to the solution to a problem.

7.6.C Analyze and compare mathematical strategies for solving problems, and select and use one or more strategies to solve a problem.

7.6.D Represent a problem situation, describe the process used to solve the problem, and verify the reasonableness of the solution.

7.6.E Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.

7.6.F Apply a previously used problem-solving strategy in a new context.

7.6.G Extract and organize mathematical information from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.

7.6.H Make and test conjectures based on data (or information) collected from explorations and experiments.

Sliding Topics from Grades 7 and 8 to Grade 6 for a Two-Year Program Leading to Algebra 1 or Mathematics 1 in Grade 8

Reorganizing Grade 6 Mathematics Standards for all Students to Begin Algebra 1 or Mathematics 1 in Grade 8

If schools structure Algebra 1 or Mathematics 1 in grade 8 for all students, the 7/8 compression presented in this document can be taught in grade 7. In addition, schools may also choose to "slide" some or all of the following selected topics from grade 7 to grade 6 in order to balance the instructional load in grades 6 and 7. Please note that this type of restructuring carries significant challenges for both students and teachers, as described earlier. Accordingly, support structures need to be in place, such as substantial time for teacher planning, teacher professional development on the mathematics content addressed in middle school, adequate instructional time for students (the equivalent of 55 to 60 minutes a day is recommended), attention to the K–5 program, etc. Following this list of "slide" topics, there are three possible clusters of topics that might be combined, with consideration of the same precautions as above.

Integers, their meaning, graphs, and operations

Core Content 7.1: Rational numbers and linear equations: 7.1.A/B/C/D

7.1.A Compare and order rational numbers using the number line, lists, and the symbols <, >, or =.

7.1.B Represent addition, subtraction, multiplication, and division of positive and negative integers visually and numerically.

7.1.C Fluently and accurately add, subtract, multiply, and divide rational numbers.

7.1.D Define and determine the absolute value of a number.

Additional Key Content 7.5.A

7.5.A Graph ordered pairs of rational numbers and determine the coordinates of a given point in the coordinate plane.

Prime factorization

Additional Key Content 7.5.B

7.5.B Write the prime factorization of whole numbers greater than 1, using exponents when appropriate.

Probability and data

Core Content 7.4: Probability and data: 7.4.A, B, C, D (without histograms)

7.4.A Represent the sample space of probability experiments in multiple ways, including tree diagrams and organized lists.

7.4.B Determine the theoretical probability of a particular event and use theoretical probability to predict experimental outcomes.

7.4.C Describe a data set using measures of center (median, mean, and mode) and variability (maximum, minimum, and range), and evaluate the suitability and limitations of using each measure for different situations.

7.4.D Construct and interpret ~~histograms,~~ stem-and-leaf plots, and circle graphs.

The following clusters of topics could be combined for compressed instruction, with the cautions described earlier:

6.1.A/6.5.C/7.1.A: Comparing and ordering rational numbers

6.1.A Compare and order non-negative fractions, decimals, and integers using the number line, lists, and the symbols <, >, or =.

6.5.C Compare and order positive and negative integers using the number line, lists, and the symbols <, >, or =.

7.1.A Compare and order rational numbers using the number line, lists, and the symbols <, >, or =.

6.1.D/6.1.F/7.1.C: Multiplying and dividing rational numbers

6.1.D Fluently and accurately multiply and divide non-negative fractions and explain the inverse relationship between multiplication and division with fractions.

6.1.F Fluently and accurately multiply and divide non-negative decimals.

7.1.C Fluently and accurately add, subtract, multiply, and divide rational numbers.

6.2.B/7.5.A: Graphing ordered pairs

6.2.B Draw a first-quadrant graph in the coordinate plane to represent information in a table or given situation.

7.5.A Graph ordered pairs of rational numbers and determine the coordinates of a given point in the coordinate plane.