Washington Comprehensive Assessment Program

Test and Item Specifications

Geometry

Office of Superintendent of Public Instruction
OSPI

The purpose of the end-of-course exams is to measure the level of mathematics proficiency that Washington students have achieved based on Washington State K-12 Mathematics Standards. In the 2008 revision, the Washington State K-12 Mathematics Standards are organized by areas of emphasis as: Core Content, Additional Key Content and Core Processes. Each area of emphasis has specific performance expectations.

Test and Item Specifications Geometry

Contents

Test Development Guidelines Geometry

The items on the Geometry End-of-Course Exam reflect the performance expectations of the Washington State Mathematics Learning Standards. The guidelines in this document assist in writing items that match the performance expectations and their associated restrictions. Restrictions are necessary to construct a valid and reliable on-demand assessment. These restrictions are not necessary in classroom based assessments.

The item writer should be familiar with all item and rubric development guidelines listed in this section as well as specific considerations listed within each area of emphasis. A style guide determines the format of items and will be applied after the items are written.

Considerations and procedures that make item development more efficient and effective include, but are not limited to, the following guidelines.

Item Development Guidelines

Standards/Performance Expectations

- Students are expected to know all content, vocabulary, and processes in grade 3-8 and Algebra 1/Integrated Mathematics 1.
- An item may assess all or part of a performance expectation.
- An item may assess one or more performance expectations within a single area of emphasis.
- An item assessing Core Processes will use mathematics from the Core Content or Additional Key Content at the grade level being assessed.
- When applicable use stem, stimulus, and prompt rules for specific performance expectations found in each area of emphasis.

General Considerations

- Any contexts used in an item should be familiar to students.
- Stimulus content will be factually correct.
- Focus on what is essential and consequential to minimize the impact of, or need for, outside knowledge.
- Stimulus should not "trick" students into choosing or developing an incorrect or ineffective response.
- Items should avoid use of "not" or "if" unless it is essential to communicate understanding of the task. Consider substituting "when" for "if".
- Items will include language that is unbiased and that will not disadvantage a particular group of students.
- Items do not contain language or representations that might offend or demean any group of students.
- Character names will be assigned from a database that is representative of the range of ethnic diversity of Washington students.

- Manipulatives are not necessary for the exam but may be made available in the classroom.

Manipulatives and Tools Allowed	Manipulatives and Tools Not Allowed
Manipulatives that are used during the assessment should not be distributed to the students but should be available in the classroom to students who elect to use them. • Straightedge (all grades) • Ruler with centimeters and inches for grade 3 (required) • Protractor or angle ruler for grade 5 (required) • Compass for Year 2 EOC only • Abacus for visually impaired/blind students using Braille edition • Tiles, algebra tiles, cubes • Base-ten pieces • Pattern blocks, geoboards, Cuisenaire rods • Judy clocks without a digital display • Glossary of Non-Mathematics Terms • Graph paper for grades 3-8 only (must be collected and shredded) *Tools that can remain on teachers' walls:* • Hundreds charts (0-99 or 1-100 only) • Number lines with whole numbers only	Because of the multitude and variety of materials available, the following list of materials that are not allowed is not exhaustive. Consider all manipulatives "Not Allowed" if they are not listed as "Allowed". The following list addresses the most commonly asked questions concerning manipulative use from the field. • Calculators for grades 3-6 • Multiplication or addition matrices • Number lines with integers, fractions, decimals, or markings of multiples, prime, and/or composite numbers • Commercially- or student-made fraction pieces, fraction templates, or fraction materials, whether labeled or unlabeled • Dictionaries or thesauruses • Patty paper or tracing paper • Dry erase boards • Highlighters

- Calculators are allowed on the grade 7, 8, and end-of-course only. Please see the calculator policy at http://www.k12.wa.us/Mathematics/CalculatorPolicy.aspx.

Vocabulary/Context

Clear Language

- Item stems and stimulus materials should be straightforward and use simple syntax.
- Stimulus should be clear and simple with a minimum of distracting or irrelevant information unless it is appropriate for the performance expectation being assessed.
- The amount of reading should be kept to a minimum so that each item is clear and precise.
- Items will clearly indicate what is expected in a response to help students focus their responses.

Vocabulary

- Use vocabulary excel sheet located at http://www.k12.wa.us/Mathematics/TestItemSpec.aspx.
- Items use language targeted to the previous grade level or lower readability, except for required mathematics terms listed in the Test and Item Specifications document.

- Items will not assess vocabulary definitions or theorems directly, but will assess conceptual understanding and application.

Notational Considerations

- Numbers, other than years, having more than three digits to the left of the decimal point will include commas to group digits as in 435,000.
- Standard measurement abbreviations may be used; however, the unit should be spelled out if any confusion is possible, e.g., "inch" rather than "in."
- Letters used as variables are always italicized.
- The symbols "×" and "•" may be used to indicate multiplication.
- Parentheses or brackets may be used as grouping symbols to indicate multiplication.
- The symbol "÷" or a horizontal fraction bar may be used to indicate division.
- Fractions will have a horizontal line separating numerator and denominator, e.g., $\frac{1}{2}$.
- Large numbers may be represented with a heading labeled "in thousands" or "in millions" in tables, charts, or graphs.
- Decimals between negative one and one are written with a leading zero, e.g., 0.25 rather than .25.
- Illustrations of figures may include hash marks on line segments to indicate congruent sides.
- Right angles will be indicated in the graphics or item stem.
- The symbol "⊥" may be used to indicate perpendicular lines, e.g., $\ell_1 \perp \ell_2$ at grade 8 MSP and end-of-course exams.
- The symbol "∥" may be used to indicate parallel lines, e.g., $\ell_1 \parallel \ell_2$ at grade MSP and end-of-course exams.
- Each graph or table will include a title in the prompt and/or the answer space.

Rules for Stimulus, Stem, and Prompt Content

- Stimulus content should be clear and simple with a minimum of distracting or irrelevant information unless it is appropriate for the performance expectation being assessed.
- Stimulus content should not "trick" students into choosing or developing an incorrect or ineffective response.
- Stimulus may include appropriate and relevant tables, charts, graphs, diagrams, and/or pictorial representations of objects, shapes, or figures.
- Items will focus on what is essential and consequential to minimize the impact of, or need for, outside knowledge.
- The amount of reading will be kept to a minimum so that each item is clear and precise.

Rules for Multiple-Choice Items

- Each Multiple-Choice item has four answer choices, the correct answer and three distractors (wrong answer choices).
- Each Multiple-Choice item will have a stem (question or statement).
- Multiple-Choice item stems will present a clear indication of what is required so that

students will know what to do before looking at the answer choices.

- The four answer choices will be approximately the same length, will have the same format, and will be syntactically and semantically parallel.
- The answer choices will be arranged in numerical or chronological order or according to length.
- Students should not be able to rule out a distractor or identify the answer simply because of superficial or trivial characteristics, syntactic complexity, or concept complexity.
- Distractors will reflect common errors or misunderstandings, naive pre-conceptions, or other misconceptions.
- Distractors will not be partially correct responses nor will they be designed to "trick" students into responding incorrectly.
- The responses "All of the above" and "None of the above" will not be used.
- The letters A, B, C, and D will be used for answer choices and will not be used as labels within a multiple-choice item in either upper or lower case.

Rules for Completion Items

- Completion items should be written like a multiple-choice item but no answer choices are provided.
- Completion items will give clear indications of what is required of students.
- Completion items will have a unique numeric answer or a number in a specified interval, i.e. answers derived using estimates of pi.
- Completion items will give a directive and reword the directive in the format of a question in the box with a line for the student's response. e.g. "Determine the quotient.", "What is the quotient?"
- Answer will not be scored for labels. Labels should be included in the question and/or answer space.

Rules for Short-Answer Items

- All EOC Short-Answer items assess content PEs in reporting strands used to calculate stu dent scores for purposes of meeting standard. Please refer to EOC Test Specifications for further information.
- Short-Answer items will give clear indications of what is required of students; e.g., "Name two properties of Figure A." or "Write an equation."
- Anything required by the scoring rubric will be asked for in the item.
- Item response spaces may be written to guide responses. A response that requires multiple parts may be scaffolded with boxes to draw attention to the parts.
- Directions with multiple requirements will be organized with bullets.
- When an item poses a specific question, the question is repeated at the bottom of the workspace with a line for the student's response.
- When an item gives a directive, then a question based on the directive will appear in the box with a line for the student's response. e.g. "Determine the surface area of the pyramid.", "What is the surface area of the pyramid?"

- General directions that allow the student to construct a response may read as follows: "Show your work using words, numbers and/or pictures." “Show the steps you used to solve the problem.”
- Any Short-Answer item that requires the student to use information from a stimulus will specifically ask for the information; e.g., “Use numbers from the table to …” or “Support your answer with information from the chart.”
- Short-Answer items may ask for a figure, diagram, equation, and/or a few sentences.
- Short Answer items will require a limited number of steps to develop a viable solution, demonstrate an understanding or process, communicate a mathematical idea or result, or show reasoning.
- Short-Answer items include Enhanced Multiple-Choice items that ask students to select from a list of four answer choices and then show work to support or explain the reason(s) for choosing that response.

Rules for Process Items

- The process PEs require a connecting content PE; e.g., 6.6.C (6.1.D). The 6.6.C indicates the item presents a sixth-grade problem-solving situation. The (6.1.D) indicates that multiplication and division of non-negative fractions is likely to be needed to solve the problem. EOC example of process PE; e.g., A1.8.B(A1.3.A)/M1.8.B(M1.2.A). The A1.8.B/M1.8.B indicates the item present an Algebra 1/Integrated Mathematics 1 problem solving situation. The A1.3.A/M1.2.A indicates that the content relates to functions and their characteristics.

Scoring Rubric Development Guidelines

- An item-specific scoring rubric will be developed for each Short-Answer item during the writing of the item.
- Short-Answer items will be scored with a 3-level scoring rubric (0-2).
- Score point elements will be based on the requirements of the item and its performance expectation(s).
- Scoring rubrics will focus on conceptual understanding, application of appropriate proce dures/strategies, and accuracy.
- Scoring rubrics will not consider conventions of writing (complete sentences, usage/gra mmar, spelling, capitals, punctuation, and paragraphing), as long as the wording of the r esponse does not interfere with the mathematical communication.
- Scoring rubrics that involve measured values will require students to label units when th e labels are not provided in the answer space.
- Scoring rubrics will be edited during pilot range finding.
- Scoring rubrics may be edited during operational range finding.

Cognitive Complexity

from Depth-of-Knowledge Levels for Four Content Areas, Norman L. Webb, March 28, 2002

"Level 1 (Recall) includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, and straight algorithmic procedure should be included at this lowest level. Other key words that signify a Level 1 include "identify," "recall," "recognize," "use," and "measure." Verbs such as "describe" and "explain" could be classified at different levels depending on what is to be described and explained.

"Level 2 (Skill/Concept) includes the engagement of some mental processing beyond a habitual response. A Level 2 assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords that generally distinguish a Level 2 item include "classify," "organize," "estimate," "make observations," "collect and display data," and "compare data." These actions imply more than one step. … Caution is warranted in interpreting Level 2 as only skills because some reviewers will interpret skills very narrowly, as primarily numerical skills, and such interpretation excludes from this level other skills such as visualization skills and probability skills, which may be more complex simply because they are less common. Other Level 2 activities include explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

"Level 3 (Strategic Thinking) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity, however, that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and using concepts to solve problems.

"Level 4 (Extended Thinking) requires complex reasoning, planning, developing, and thinking most likely over an extended period of time." (This level is best assessed at the classroom level and not relevant to this on-demand assessment.)

Geometry

G.1. Core Content: *Logical arguments and proofs* *(Logic)*
Students formalize the reasoning skills they have developed in previous grades and solidify their understanding of what it means to prove a geometric statement mathematically. In Geometry, students encounter the concept of formal proof built on definitions, axioms, and theorems. They use inductive reasoning to test conjectures about geometric relationships and use deductive reasoning to prove or disprove their conclusions. Students defend their reasoning using precise mathematical language and symbols.

G.2 Core Content*: Lines and Angles* *(Geometry/Measurement)*
Students study basic properties of parallel and perpendicular lines, their respective slopes, and the properties of the angles formed when parallel lines are intersected by a transversal. They prove related theorems and apply them to solve both mathematical and practical problems.

G.3. Core Content: *Two- and three-dimensional figures* *(Geometry/Measurement)*
Students know and can prove theorems about two- and three-dimensional geometric figures, both formally and informally. They identify necessary and sufficient conditions for proving congruence, similarity, and properties of figures. Triangles are a primary focus, beginning with general properties of triangles, working with right triangles and special triangles, proving and applying the Pythagorean Theorem and its converse, and applying the basic trigonometric ratios of sine, cosine, and tangent. Students extend their learning to other polygons and the circle, and do some work with three-dimensional figures.

G.4. Core Content*: Geometry in the coordinate plane* *(Geometry/Measurement, Algebra)*
Students make connections between geometry and algebra by studying geometric properties and attributes that can be represented on the coordinate plane. They use the coordinate plane to represent situations that are both purely mathematical and that arise in applied contexts. In this way, they use the power of algebra to solve problems about shapes and space.

G.5. Core Content: *Geometric transformations* *(Geometry/Measurement)*
Students continue their study of geometric transformations, focusing on the effect of such transformations and the composition of transformations on the attributes of geometric figures. They study techniques for establishing congruence and similarity by means of transformations.

G.6. Additional Key Content *(Measurement)*
Students extend and formalize their work with geometric formulas for perimeter, area, surface area, and volume of two- and three-dimensional figures, focusing on mathematical derivations of these formulas and their applications in complex problems. They use properties of geometry and measurement to solve problems in purely mathematical as well as applied contexts. Students understand the role of units in measurement and apply what they know to solve problems involving derived measures like speed or density. They understand that all measurement is approximate and specify precision in measurement problems.

G.7. Core Processes*: Reasoning, problem solving, and communication*

Students formalize the development of reasoning in Geometry as they become more sophisticated in their ability to reason inductively and begin to use deductive reasoning in formal proofs. They extend the problem-solving practices developed in earlier grades and apply them to more challenging problems, including problems related to mathematical and applied situations. Students use a coherent problem-solving process in which they analyze the situation to determine the question(s) to be answered, synthesize given information, and identify implicit and explicit assumptions that have been made. They examine their solution(s) to determine reasonableness, accuracy, and meaning in the context of the original problem. They use correct mathematical language, terms, symbols, and conventions as they address problems in Geometry and provide descriptions and justifications of solution processes. The mathematical thinking, reasoning, and problem-solving processes students learn in high school mathematics can be used throughout their lives as they deal with a world in which an increasing amount of information is presented in quantitative ways, and more and more occupations and fields of study rely on mathematics.

Test Organization Geometry

The Geometry End-of-Course Exam and Retake Year 2 will contain 37 items that assess the perfo rmance expectations common to Geometry/Integrated Mathematics 2. These items are used to determine a student's scale score for purposes of graduation. The performance expectations co mmon to Geometry/Integrated Mathematics 2 will be assessed using multiple-choice, completi on, and short-answer items on the end-of-course exams and retake exams.

In addition, the Geometry End-of-Course Exam will contain 6 items that assess the performance expectations common to Geometry/Integrated Mathematics 1 and Geometry/Integrated Mathematics 3. These performance expectations are assessed and reported but are not incorporated into a student's scale score for purposes of graduation. These performance expectations will be assessed using multiple-choice and completion items on end-of-course exams only.

Multiple-Choice Items	Completion Items	Short-Answer Items
• Each Multiple-Choice ite m has four answer choic es, the correct answer a nd three distractors. • Multiple choice items ar e worth one point each. • There will be 29 Multipl e-Choice items assessing PEs common to Geomet ry/Integrated Mathemat ics 2. • There will be 3-5 Multipl e-Choice items assessing PEs common to Geomet ry/Integrated Mathemat ics 1 and Geometry/Inte grated Mathematics 3. NOTE: Enhanced Multiple-Choice items are scored as Short-Answer items.	• Each Completion item re quires the student to en ter a numerical answer, an expression with varia bles, or an equation wit h variables. • Completion items are w orth one point each. • There will be 5 Completi on items assessing PEs c ommon to Geometry/In tegrated Mathematics 2 . • There will be 1-3 Compl etion items assessing PE s common to Geometry/ Integrated Mathematics 1 and Geometry/Integra ted Mathematics 3.	• Each Short-Answer item requires a constructe d response. • A Short-Answer item may ask the student to write a sentence or equation; complete a tabl e, graph, or chart; draw a picture; construct a diagram; or perform a calculation. • An Enhanced Multiple-Choice item will ask th e student to select from a list of four answer c hoices and then show work to support or expl ain the reason(s) for choosing that answer or to solve a problem. No more than two items on a test will be Enhanced Multiple-Choice ite ms. • Short-Answer items are worth two points eac h. • There will be 3 Short-Answer items assessing PEs common to Geometry/Integrated Mathe matics 2. • Short-Answer items will not assess PEs comm on to I Geometry/Integrated Mathematics 1 o r Geometry/Integrated Mathematics 3.

The Geometry End-of-Course Exam is intended to be administered to intact classrooms over three days.

OSPI's recommendations on administering the EOCs are as follows:

1. The EOCs should be given to intact classrooms over three days (30 minutes for directions/distributing materials, 120 minutes of testing time).
2. Students are expected to finish within 120 minutes of testing time.

Tools, including approved calculators, are allowed for the test administration. Please refer to t he calculator policy for more information.

Geometry End-of-Course Test Map

Reporting Strands	Number of Items				Total Number of Points
	MC	CP	SA	Total	
Logical arguments and proof*	5-8	0	0-1	6-8	6-8
Proving and applying properties of 2-dimensional figures *	15-19	2-4	1-3	21-24	24-26
Figures in a coordinate plane and measurement *	5-8	1-3	0-1	7-9	7-9
Total Number of Items used to determine scale score**	29	5	3	37	
Total Number of Points used to determine scale score**	29	5	6		40
Course-Specific content***	3-5	1-3	0	6	6
Total Number of Items				43	

**Items assessing these reporting strands assess performance expectations common to Geometry/Integrated Mathematics 2 and are used to calculate student scores for purposes of meeting standard.*

***A scale score on the end-of-course exam is used to determine a student's proficiency level: below basic, basic, proficient, advanced.*

****Items assessing course-specific content are in addition to the 37 items used to determine a student's scale score. Student performance on these items is reported but is not incorporated into student scale scores for purposes of meeting standard.*

Geometry Performance Expectations by Reporting Strand

Reporting Strands	Performance Expectations	
	Content	Process**
Logical arguments and proof	G.1.D G.1.E G.1.F	G.7.A G.7.B G.7.C G.7.E G.7.G
Proving and applying properties of 2-dimensional figures	G.3.A G.3.B* G.3.C G.3.D G.3.E G.3.F G.3.G	G.7.A G.7.B G.7.C G.7.E G.7.G
Figures in a coordinate plane and measurement	G.4.B G.4.C G.6.E G.6.F	G.7.A G.7.B G.7.C G.7.E G.7.G
Course-Specific content	G.1.A G.2.A G.2.B G.2.C G.2.D G.3.B* G.3.H G.3.I G.3.J G.3.K G.4.A G.4.D G.5.A G.5.B G.5.C G.5.D G.6.A G.6.C G.6.D	

** G.3.B includes triangle similarity and triangle congruence. Triangle similarity aligns with Integrated Mathematics 1 and will be assessed as Course-specific Content. Triangle congruence aligns with Integrated Mathematics 2 and will be assessed in* ***Proving and applying properties of 2-dimensional figures****.*

*** Items assessing process performance expectations (PEs) must include content at the Geometry level. The content of items assessing process PEs determines its reporting strand.*

Item Specifications Geometry

Item specifications for each Area of Emphasis are organized in two sections:

Stimulus, Stem, and Prompt Rules

Stimulus, stem, and prompt rules list area-specific guidelines for developing items. The rules are in addition to those included in the Item Development Guidelines.

Content Expectations

The performance expectations in this document are identical to those in the Washington State Geometry Mathematics Standards. Performance expectations that will be assessed at the state level appear in **bold text**. The remaining performance expectations, which appear in *italicized text*, should be taught and assessed at the classroom level.

Items assessing G.7 Core Processes will use Core Content or Additional Key Content performance expectations from Geometry.

The information in the columns to the right of each performance expectation shows item development parameters for:

- Cognitive complexity (C.C.) as Level 1, Level 2, Level 3, or Level 4 as defined by Norman Webb.
- Format as Multiple-Choice (MC), Completion (CP), or Short-Answer (SA).
- Contextual Situation (Ctxt) as required (Y), item dependent (I), or not allowed (N).

G.1 Core Content: Logical arguments and proofs

(Logic)

Stimulus, Stem, and Prompt Rules

- Use Item Development Guidelines at the beginning of this document.
- Items assessing G.1.D will not list the terms converse, inverse and contrapositive as answer options.

Content Expectations

Items may ask students to:	*Integrated Sequence*		*C.C.*	*Format*	*Ctxt*
G.1.A Distinguish between inductive and deductive reasoning.	**M1.4.A**		2	MC	I
G.1.B *Use inductive reasoning to make conjectures, to test the plausibility of a geometric statement, and to help find a counterexample*.	M1.4.B		(*2,3*)	NA	NA
G.1.C Use deductive reasoning to prove that a valid geometric statement is true.	**M1.4.C**	**M2.3.A**	2,3	*	*
G.1.D Write the converse, inverse, and contrapositive of a valid proposition and determine their validity.		**M2.3.C**	2,3	MC,SA	Y
G.1.E Identify errors or gaps in a mathematical argument and develop counterexamples to refute invalid statements about geometric relationships.		**M2.3.B**	2,3	MC	I
G.1.F *Distinguish between definitions and undefined geometric terms and* **explain the role of definitions, undefined terms, postulates (axioms), and theorems**.		**M2.3.D**	1	MC	N

* This performance expectation may be included in items assessing core process performance expectations or performance expectations that ask students to "know, prove, and apply" mathematical content.

G.2 Core Content: Lines and angles

(Geometry/Measurement)

Stimulus, Stem, and Prompt Rules

- Use Item Development Guidelines at the beginning of this document.
- Items assessing G.2 may require students to apply the theorems listed in the Theorems s ection at the end of this document.
- Items assessing G.2 may include figures formed by parallel lines, perpendicular lines, an d transversals.
- Items assessing G.2 may expect students to apply theorems to prove geometric stateme nts, not to prove theorems.
- Items assessing G.2 may expect students to complete a partial proof presented in two-c olumn format, flow chart, or paragraph.
- Items assessing G.2.C will ask students to identify visuals representing the steps used in constructions related to parallel and perpendicular lines.

Content Expectations

Items may ask students to:		*Integrated Sequence*		*C.C.*	*Format*	*Ctxt*
G.2.A	**Know, prove, and apply theorems about parallel and perpendicular lines.**	**M1.4.E**		2,3	MC,CP	I
G.2.B	**Know, prove, and apply theorems about angles, including angles that arise from parallel lines intersected by a transversal.**	**M1.4.F**		2,3	MC,CP	I
G.2.C	*Explain and* **perform basic compass and straightedge constructions related to parallel and perpendicular lines.**	**M1.4.G**		1,2	MC	N
G.2.D	**Describe the intersections of lines in the plane and in space, of lines and planes, and of planes in space.**	**M3.5.A**		1	MC	N

Key: ***Format***= Multiple-Choice (MC), Completion (CP), Short-Answer (SA), or Not Assessed (NA)
C.C.= Cognitive Complexity **(#)** = Cognitive Complexity for italicized text
Ctxt=Contextual Situation ***I***= Item dependent ***Y***= Yes Context ***N***= No Context

G.3 Core Content: Two- and three-dimensional figures

(Geometry/Measurement)

Stimulus, Stem, and Prompt Rules

- Use Item Development Guidelines at the beginning of this document.
- Items assessing G.3 may require students to apply the theorems listed in the Theorems s ection at the end of this document.
- Items assessing G.3 may include information about and require the use of exterior angle s.
- Items assessing G.3 may include oblique three-dimensional figures.
- Items assessing G.3 may expect students to apply theorems to prove geometric stateme nts, not to prove theorems.
- Items assessing G.3 may expect students to complete a partial proof presented in two-c olumn format, flow chart, or paragraph.
- Items assessing G.3.A may include similar triangles formed by a line segment parallel to one side of the triangle.
- Items assessing G.3.A will not list the terms centroid, othocenter, circumcenter, and ince nter as answer options.
- Short-Answer items assessing G.3.B will not include triangle similarity.
- Items assessing G.3.D may include figures that can be broken down into right triangles a nd other polygons.
- Items assessing G.3.E may expect students to determine and/or label side lengths and/o r angle measures of a right triangle.
- Multiple-Choice Items assessing G.3.E may present answer choices in terms of sine, cosi ne, or tangent.
- Items assessing G.3.I will ask students to identify visuals representing the steps used in c onstructions related to the circle.

Content Expectations

Items may ask students to:		*Integrated Sequence*		*C.C.*	*Format*	*Ctxt*
G.3.A	**Know,** ***explain,*** **and apply basic postulates and theorems about triangles and the special lines, line segments, and rays associated with a triangle.**		**M2.3.E**	2	MC,SA	N
G.3.B	**Determine and prove triangle congruence, triangle similarity, and other properties of triangles.**	**M1.4.D**	**M2.3.F**	2,3	MC,SA	N
G.3.C	**Use the properties of special right triangles (30°–60°–90° and 45°–45°–90°) to solve problems.**		**M2.3.I**	1,2	MC,CP	I
G.3.D	**Know,** ***prove,*** **and apply the Pythagorean Theorem and its converse.**		**M2.3.G**	1,2	MC,CP	I

Performance Expectation G.3.E through G.3.K on the next page.

Key: ***Format***= Multiple-Choice (MC), Completion (CP), Short-Answer (SA), or Not Assessed (NA)
C.C.= Cognitive Complexity **(#)** = Cognitive Complexity for italicized text
Ctxt=Contextual Situation ***I***= Item dependent ***Y***= Yes Context ***N***= No Context

Items may ask students to:		*Integrated Sequence*		*C.C.*	*Format*	*Ctxt*
G.3.E	**Solve problems involving the basic trigonometric ratios of sine, cosine, and tangent.**		**M2.3.H**	1,2	MC,SA	I
G.3.F	**Know, prove, and apply basic theorems about parallelograms.**		**M2.3.J**	2,3	MC,CP	I
G.3.G	**Know, prove, and apply theorems about properties of quadrilaterals and other polygons.**		**M2.3.K**	2,3	MC,CP	I
G.3.H	**Know, prove, and apply basic theorems relating circles to tangents, chords, radii, secants, and inscribed angles.**	**M3.7.A**		2,3	MC,CP	I
G.3.I	*Explain* **and perform constructions related to the circle.**	**M3.7.C**		1,2	MC	N
G.3.J	**Describe prisms, pyramids, parallelepipeds, tetrahedra, and regular polyhedra in terms of their faces, edges, vertices, and properties.**	**M3.5.B**		1	MC	N
G.3.K	**Analyze cross-sections of cubes, prisms, pyramids, and spheres and identify the resulting shapes.**	**M3.5.C**		1,2	MC	N

Key: ***Format***= Multiple-Choice (MC), Completion (CP), Short-Answer (SA), or Not Assessed (NA)
C.C.= Cognitive Complexity **(#)** = Cognitive Complexity for italicized text
Ctxt=Contextual Situation ***I***= Item dependent ***Y***= Yes Context ***N***= No Context

G.4 Core Content: Geometry in the coordinate plane

(Geometry/Measurement, Algebra)

Stimulus, Stem, and Prompt Rules

- Use Item Development Guidelines at the beginning of this document.
- Items assessing G.4. may include a blank grid; only the answer choice (Multiple-Choice It ems) or the student answer (Completion Item) is scored.
- Items assessing G.4.A may represent the equation of a line in standard form, point-slope form, or slope-intercept form
- Items assessing G.4.B may expect students to determine the coordinates of a midpoint o f a line segment or a vertex of a figure.
- Items assessing G.4.B may expect students to represent the coordinates of a point in ter ms of the coordinates of other points.
- Items assessing G.4.B may include a diagram on a coordinate plane.
- Items assessing G.4.C may expect students to name triangles or quadrilaterals based on angle types, side lengths, and side relationships.
- Items assessing G.4.D may expect students to determine the equation for a circle given t he center and radius or the center and a point on the circle.
- Items assessing G.4.D may expect students to determine the center or radius of a circle given the equation for the circle.
- The equation for a circle will be presented in the form $(x-h)^2+\ (y-k)^2=\ r^2$.

Content Expectations

Items may ask students to:		*Integrated Sequence*		*C.C.*	*Format*	*Ctxt*
G.4.A	**Determine the equation of a line in the coordinate plane that is described geometrically, including a line through two given points, a line through a given point parallel to a given line, and a line through a given point perpendicular to a given line.**	**M1.3.H**		1,2	MC,CP	N
G.4.B	**Determine the coordinates of a point that is described geometrically**.		**M2.3.L**	1,2	MC,CP	I
G.4.C	*Verify and* **apply properties of triangles and quadrilaterals in the coordinate plane.**		**M2.3.M**	1,2	MC, SA	I
G.4.D	**Determine the equation of a circle that is described geometrically in the coordinate plane and, given equations for a circle and a line, determine the coordinates of their intersection(s).**	**M3.7.B**		1,2	MC,CP	N

Key: ***Format***= Multiple-Choice (MC), Completion (CP), Short-Answer (SA), or Not Assessed (NA)
C.C.= Cognitive Complexity **(#)** = Cognitive Complexity for italicized text
Ctxt=Contextual Situation ***I***= Item dependent ***Y***= Yes Context ***N***= No Context

G.5 Core Content: Geometric transformations

(Geometry/Measurement)

Stimulus, Stem, and Prompt Rules

- Use Item Development Guidelines at the beginning of this document.
- Items assessing G.5.A may expect students to identify the results of transformations and composition of transformations.
- Items assessing G.5.A may present a rule for performing a transformation in the form (x, y) → ($ax+h$, $by+k$) or (x,y) → ($ay+h$, $bx+k$).
- Items may include compositions, inverses, commutativity, or associativity of transformat ions.
- Items may expect students to identify the congruence or similarity of figures under vario us transformations.
- Items assessing G.5.B and G.5.C may include line segments or polygons.
- Dilations will be centered at the origin, an endpoint or midpoint of a segment, or a verte x of a polygon.

Content Expectations

Items may ask students to:		*Integrated Sequence*	*C.C.*	*Format*	*Ctxt*
G.5.A	**Sketch results of transformations and compositions of transformations for a given two-dimensional figure on the coordinate plane, and describe the rule(s) for performing translations or for performing reflections about the coordinate axes or the line $y = x$.**	**M3.2.A**	2	MC	I
G.5.B	**Determine and apply properties of transformations.**	**M3.2.B**	2	MC	N
G.5.C	**Given two congruent or similar figures in a coordinate plane, describe a composition of translations, reflections, rotations, and dilations that superimposes one figure on the other.**	**M3.2.C**	2	MC	N
G.5.D	**Describe the symmetries of two-dimensional figures and describe transformations, including reflections across a line and rotations about a point.**	**M3.2.D**	2	MC,CP	I

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G.6 Additional Key Content

(Measurement)

Stimulus, Stem, and Prompt Rules

- Use Item Development Guidelines at the beginning of this document.
- Items assessing G.6.A may expect students to determine measures of a circle (radius, dia meter, circumference, area) given arc length or area of a sector.
- Items assessing G.6.C may include figures that can be broken down into cones, pyramids , cylinders, and prisms.
- Items assessing G.6.C may include oblique and right prisms, oblique and right cylinders, r ight pyramids, right cones, and spheres.
- Items assessing G.6.D may include polygons, cones, pyramids, cylinders, and prisms.
- Items assessing G.6.E will not ask students to estimate a calculation.
- Items assessing G.6.F may include area measurements, volume measurements, and deri ved units of measure such as speed, density, flow rates, or population density.
- Items assessing G.6.F that ask students to convert between systems will provide a conve rsion factor.

Content Expectations

Items may ask students to:		*Integrated Sequence*		*C.C.*	*Format*	*Ctxt*
G.6.A	*Derive and* **apply formulas for arc length and area of a sector of a circle.**	**M3.7.D**		1,2	MC	I
G.6.B	*Analyze distance and angle measures on a sphere and apply these measurements to the geometry of the earth.*	M3.5.F		(*1*)	NA	NA
G.6.C	**Apply formulas for surface area and volume of three-dimensional figures to solve problems.**	**M3.5.D**		1,2	MC,CP	I
G.6.D	*Predict and* **verify the effect that changing one, two, or three linear dimensions has on perimeter, area, volume, or surface area of two- and three-dimensional figures**.	**M3.5.E**		2	MC,CP	I
G.6.E	**Use different degrees of precision in measurement, explain the reason for using a certain degree of precision, and apply estimation strategies to obtain reasonable measurements with appropriate precision for a given purpose.**		**M2.5.B**	2	MC,SA	I

Performance Expectation G.6.F on the next page.

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Items may ask students to:		*Integrated Sequence*		*C.C.*	*Format*	*Ctxt*
G.6.F	**Solve problems involving measurement conversions within and between systems, including those involving derived units, and analyze solutions in terms of reasonableness of solutions and appropriate units.**		**M2.5.C**	2	MC,CP	I

Key: ***Format***= Multiple-Choice (MC), Completion (CP), Short-Answer (SA), or Not Assessed (NA)
C.C.= Cognitive Complexity **(#)** = Cognitive Complexity for italicized text
Ctxt=Contextual Situation ***I***= Item dependent ***Y***= Yes Context ***N***= No Context

G.7 Core Processes: Reasoning, problem solving, and communication

Stimulus, Stem, and Prompt Rules

- Use Item Development Guidelines at the beginning of this document.
- Problem solving items will include G.7.A, G.7.B, G.7.C.
- Items assessing G.7.A-C may include figures that are not drawn to scale, include a statement "Picture is **not** drawn to scale."
- Items assessing G.7.G may ask a student to support or contradict a given conclusion.
- Mathematics content for process items must be from performance expectations that are common to Geometry and Mathematics 2.

Content Expectations

Items may ask students to:	*Integrated Sequence*		*C.C.*	*Format*	*Ctxt*
G.7.A Analyze a problem situation and represent it mathematically.		**M2.6. A-C**	3	MC,SA	I
G.7.B Select and apply strategies to solve problems.					
G.7.C Evaluate a solution for reasonableness, verify its accuracy, and interpret the solution in the context of the original problem.					
G.7.D *Generalize a solution strategy for a single problem to a class of related problems, and apply a strategy for a class of related problems to solve specific problems.*		M2.6.D	(*3,4*)	NA	NA
G.7.E Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.		**M2.6.E**	2,3	SA	I
G.7.F *Summarize mathematical ideas with precision and efficiency for a given audience and purpose.*		M2.6.F	(*2,3*)	NA	NA
G.7.G Synthesize information to draw conclusions and evaluate the arguments and conclusions of others.		**M2.6.G**	3	MC,SA	I
G.7.H *Use inductive reasoning to make conjectures, and use deductive reasoning to prove or disprove conjectures.*		M2.6.H	(*4*)	NA	NA

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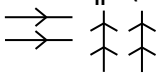
C.C.= Cognitive Complexity **(#)** = Cognitive Complexity for italicized text
Ctxt=Contextual Situation ***I***= Item dependent ***Y***= Yes Context ***N***= No Context

Mathematics Symbols First Used in Exam Items

Strand	Name	Symbol	Grade
Operations	addition	+	3
	subtraction	-	3
	multiplication	×	3
		( )	5
		•	6
	division	÷	3
		– (for ex. $\frac{6}{3}$)	4
	absolute value of *a*	$\|a\|$	7
	exponents	base exponent (for ex. 3^2)	8
	square root of *a*	$\sqrt{a}$	8
Algebra	equal to	=	3
	less than	<	3
	greater than	>	3
	less than or equal to	≤	8
	greater than or equal to	≥	8
	not equal to	≠	8
	function	f(*x*)	EOC
	brackets	[ ]	EOC
Geometric Sense	right angle	 (in diagram)	3
	pi	π	6
	congruent	 (in diagram)	7
	line segment AB	$\overline{AB}$	7
	angle A	$\angle A, \angle BAC$	8
	triangle ABC	$\Delta\, ABC$	8
	perpendicular	⊥ (in text)	8
	parallel	∥ (in text)	8
		 (in diagram)	EOC
	similar	~	EOC
	measure of angle A	$m \angle A$	EOC
	congruent	≅	EOC
	ray AB	$\overrightarrow{AB}$	EOC
	line AB	$\overleftrightarrow{AB}$	EOC
	arc AB	$\overset{\frown}{AB}$	EOC

Mathematics Formula Sheets for End-of-Course Exams

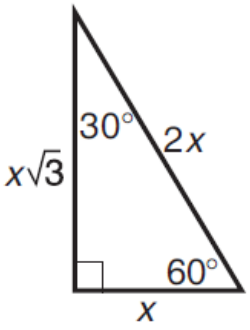
Use at least two decimal place values when approximating square roots or trigonometric ratios.

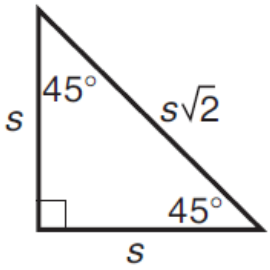
Description	Formula	Variables
Arc Length	$L = \frac{m\overset{\frown}{BC}}{360^\circ}\pi d$	*L*: Arc Length *B, C*: endpoints of arc *d*: diameter of the circle *m*: the measure of
Area of Sector	$A = \frac{m\overset{\frown}{BC}}{360^\circ}\pi r^2$	*A*: Area of Sector *B, C*: endpoints of intercepted arc *r*: radius of the circle *m*: the measure of
Cylinder	$SA = 2\pi r^2 + 2\pi rh$	*SA*: Surface Area *r*: radius of the base *h*: height
	$V = \pi r^2 h$	*V*: Volume *r*: radius of the base *h*: height
Cone	$SA = \pi r^2 + \pi rl$	*SA*: Surface Area *r*: radius of the base *l*: slant height
	$V = \frac{1}{3}Bh$ **or** $V = \frac{1}{3}\pi r^2 h$	*V*: Volume *r*: radius of the base *h*: height *B*: area of the base
Prism	$V = Bh$	*V*: Volume *B*: area of the base *H*: height
	$SA = 2B + Ph$ **or** $SA = 2B + L$	*SA*: Surface Area *B*: area of the base *P*: Perimeter of the base *h*: height *L*: lateral surface area
Pyramid	$V = \frac{1}{3}Bh$	*V*: Volume *B*: area of the base *h*: height
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	*x*: solution a, b, c: coefficients
Sphere	$V = \frac{4}{3}\pi r^3$	*V*: Volume *r*: radius
	$SA = 4\pi r^2$	*SA*: Surface Area *r*: radius

Mathematics Formula Sheets for End-of-Course Exams

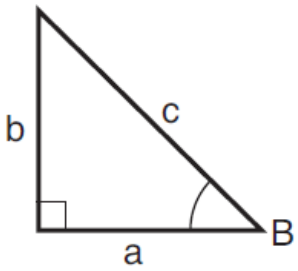
Use at least two decimal place values when approximating square roots or trigonometric ratios.

Special Right Triangles





Trigonometric Ratios

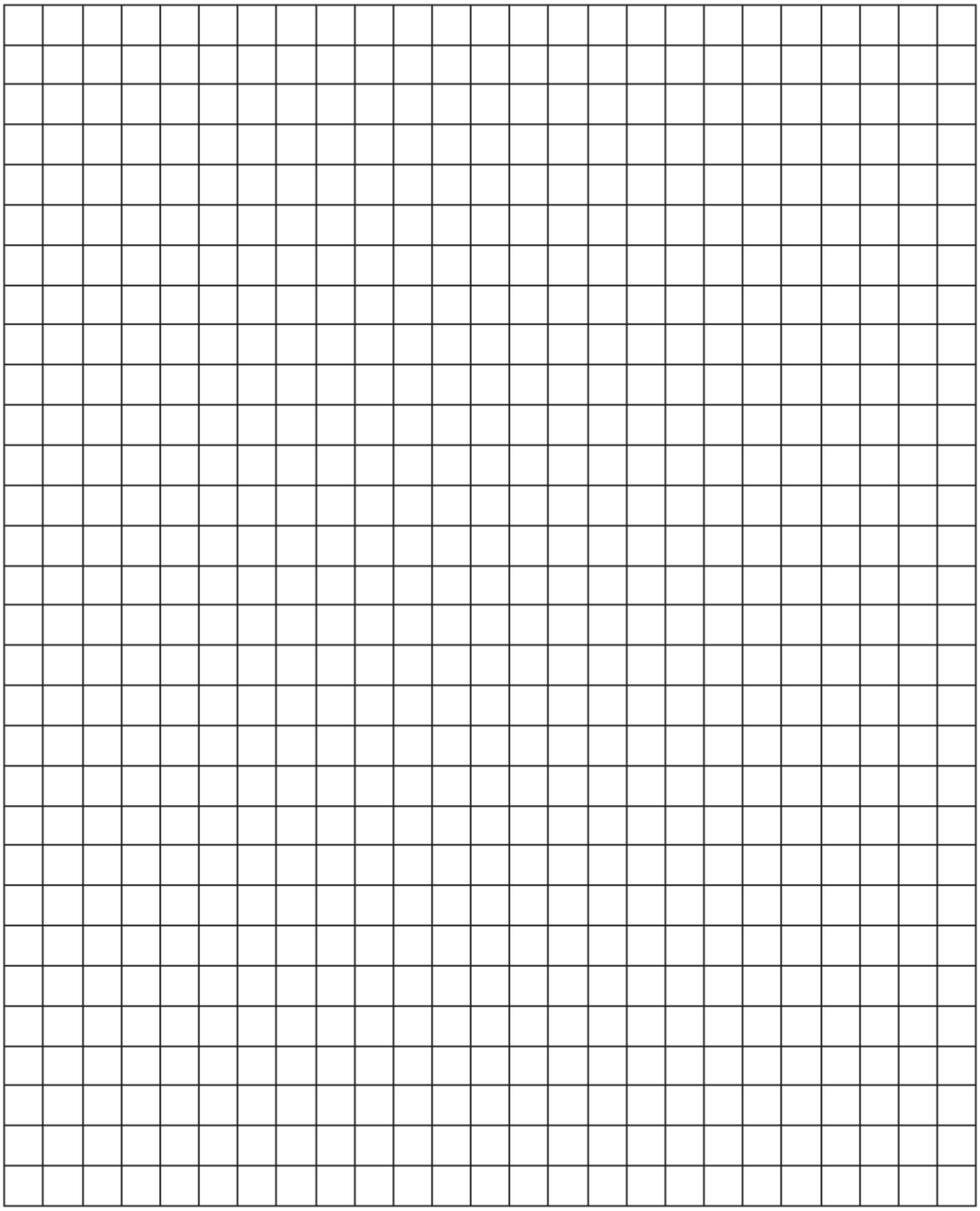


$$\sin B = \frac{b}{c}$$

$$\cos B = \frac{a}{c}$$

$$\tan B = \frac{b}{a}$$

Mathematics End-of-Course Graph Paper



Measurement Vocabulary

Attributes, Units, Abbreviations, and Grade Level First-Used in Assessment Items

The levels in parentheses refer to the grades at which students should develop an oral understanding of the terms according to the K-2 Standards.

Attribute	Unit	Grade
Length (K)	inch (in.)	3 (2)
	foot (ft)	3 (2)
	yard (yd)	3 (2)
	mile (mi)	4
	millimeter (mm)	4
	centimeter (cm)	3 (2)
	meter (m)	3 (2)
	kilometer (km)	4
Capacity (K)	cup (c)	3
	pint (pt)	3
	quart (qt)	3
	gallon (gal)	3
	milliliter (mL)	3
	liter (L)	3
	kiloliter (kL)	4
Weight (K)	ounce (oz)	3
	pound (lb)	3
	ton (t)	7
Mass	gram (g)	3
	kilogram (kg)	4

Attribute	Unit	Grade
Time	second (s)	4
	minute (min)	4 (2)
	hour (h)	4 (2)
	day (d)	4 (2)
	week (wk)	4 (2)
	month (mo)	4 (2)
	year (y)	4 (2)
Temperature	Degree Fahrenheit (^oF)	3
	Degree Celsius (^oC)	3
Angle	Degrees (^{o})	5

Conversions

- In grades 4-6, students are expected to convert within a measurement system but not between measurement systems. For example, 3 feet = 1 yard is a conversion within the U.S. customary system, but 1 yard ≈ 0.91 meter is a conversion between U.S. Customary and metric systems.
- In grade 7 and Geometry/Integrated Mathematics 2 EOCs students may be asked to convert between systems and a conversion factor will be provided.
- Most dictionaries list conversion factors.

This list of definitions, postulates, and theorems describes geometric concepts students are expected to know, understand and apply. Students are not expected to know or memorize the exact wording of the items in this list. This list will not be provided during the test.

Definitions, Postulates, and Theorems

G.2.A: Know, prove, and apply theorems about parallel and perpendicular lines.

1. Given a line and a point not on the line, there exists exactly one line through the point and parallel to the given line.
2. Given a line and a point not on the line, there exists exactly one line through the point and perpendicular to the given line.
3. Given two lines, they are perpendicular if and only if their intersection forms right angles.
4. If two lines are both parallel to a third line, then they are parallel to each other.
5. If two lines are perpendicular to a third line, then they are parallel to each other.
6. If a line is perpendicular to one of two parallel lines, then it is perpendicular to the other line.
7. Two nonvertical lines in a coordinate plane are parallel if and only if they have the same slope.
8. Two nonvertical lines in a coordinate plane are perpendicular if and only if the product of their slopes is -1.
9. A point lies on the perpendicular bisector of a segment if and only if it is equidistant from the endpoints of the segment.

G.2.B: Know, prove, and apply theorems about angles, including angles that arise from parallel lines intersected by a transversal.

1. Given two lines cut by a transversal, the lines are parallel if and only if one of these angle pairs are congruent: corresponding angles, alternate interior, alternate exterior.
2. Given two lines cut by a transversal, the lines are parallel if and only if the pairs of interior or exterior angles on the same side of the transversal are supplementary.
3. If two angles are complements or supplements of the same angle or congruent angles, then the angles are congruent.
4. A point in the interior of an angle lies on the angle bisector if and only if it is equidistant from the sides of the angle.

G.2.C: *Explain and* perform basic compass and straightedge constructions related to parallel and perpendicular lines.

1. Construct a line parallel to a given line through a point not on the given line.
2. Construct a line perpendicular to a given line through a point on the given line.
3. Construct a line perpendicular to a given line through a point not on the given line.
4. Construct a line perpendicular to a given ray through the endpoint of the ray.
5. Construct a perpendicular bisector of a line segment.

G.3.A: Know, explain, and apply basic postulates and theorems about triangles and the special lines, line segments, and rays associated with a triangle.

1. A triangle is isosceles if and only if the base angles are congruent.
2. The medians of a triangle are concurrent at the centroid.
3. The lines containing the altitudes of a triangle are concurrent at the orthocenter.
4. The lines containing the perpendicular bisector of a triangle are concurrent at the circumcenter. Their common point is equidistant from the three vertices of the triangle.
5. The lines containing the angle bisectors of a triangle are concurrent at the incenter. Their common point is equidistant from the three sides of the triangle.
6. The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote angles.
7. The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
8. In a triangle, the longest side is opposite the largest angle.
9. In a triangle, the largest angle is opposite the longest side.

G.3.B: Determine and prove triangle congruence, triangle similarity, and other properties of triangles.

1. If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar. (Angle – Angle Similarity)
2. If three sides of one triangle are congruent to three sides of another triangle, then the two triangles are congruent. (Side – Side – Side Congruence)
3. If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent. (Side – Angle – Side Congruence)
4. If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent. (Angle – Side – Angle Congruence)
5. If two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of another triangle, then the two triangles are congruent. (Angle – Angle – Side Congruence)

G.3.F: Know, prove, and apply basic theorems about parallelograms.

1. A quadrilateral is a parallelogram if and only if its opposite sides are congruent.
2. A quadrilateral is a parallelogram if and only if its opposite angles are congruent.
3. A quadrilateral is a parallelogram if and only if its consecutive interior angles are supplementary.
4. A quadrilateral is a parallelogram if and only if its diagonals bisect each other.
5. A parallelogram is a rectangle if and only if its diagonals are congruent.
6. A parallelogram is a rhombus if and only if its diagonals are perpendicular.
7. A parallelogram is a rhombus if and only if its diagonals bisect pairs of opposite angles.

G.3.G: Know, prove, and apply theorems about properties of quadrilaterals and other polygons.

1. If a figure is a trapezoid, then consecutive angles between a pair of parallel lines are supplementary.
2. If a figure is a kite, then the diagonals are perpendicular.
3. The sum of one set of exterior angles of a polygon is 360^{o}.
4. The sum of the interior angles of a polygon is $((n-2)180)^{o}$ where n is the number of sides of the polygon.

G.3.H: Know, prove, and apply basic theorems relating circles to tangents, chords, radii, secants, and inscribed angles.

1. Given two congruent circles (or the same circle), two arcs are congruent if and only if their central angles are congruent.
2. Given two congruent circles (or the same circle), two chords are congruent if and only if they are equidistant from the center of the circle.
3. Given two congruent circles (or the same circle), two minor arcs are congruent if and only if their corresponding chords are congruent.
4. If the diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and the diameter bisects the arc intercepted by the chord.
5. If two inscribed angles in a circle intercept the same arc, then they have the same measure.
6. If two secants intersect in the interior of a circle, then the sum of the measures of vertical angles formed is equal to the sum of the measures of the corresponding intercepted arcs.
7. A line is tangent to a circle if and only if it is perpendicular to the radius drawn to the point of tangency.
8. An angle inscribed in a circle is a right angle if and only if its corresponding arc is a semicircle and the longest side of a resulting triangle is a diameter of the circle.
9. The measure of an inscribed angle in a circle is half the measure of the intercepted arc.
10. The measure of a central angle is equal to the measure of the intercepted arc.

G.3.I *Explain* and perform constructions related to the circle.

1. Construct the circumscribed circle for a given triangle.
2. Construct the inscribed circle for a given triangle.
3. Construct a diameter of a given circle.
4. Locate the center of a given circle.
5. Construct a line tangent to a given circle through a given point on the circle.
6. Construct lines tangent to a given circle through a given point outside the circle.