2011 Lessons Learned from Scoring Student Work On End-of-Course Exams

Rangefinding, scoring and data review of items on the Mathematics End-of-Course (EOC) Exams provide the opportunity to see hundreds of student responses at each grade and/or course level and to evaluate data summarizing student performance. The Mathematics Assessment Team would like to share observations about student responses and areas of mathematics where students appear to be struggling. Lessons Learned from Scoring Student Work lists actions students could take to increase their scores on the state assessments. Because new standards are being assessed, we have also added descriptions of content that was particularly difficult for students. Grades and/or courses will be listed separately.

In general, students fail to earn points toward a better score because of incomplete responses or incomplete mathematical representations. Students could improve their scores by:

- answering the question or completing each task in the prompt;
- using bullets as a checklist to make sure the response is complete;
- checking to see that they transcribe the correct numbers in the prompt and checking their computations for accuracy;

Information from Lessons Learned should be modeled, practiced and used throughout the school year. Students should be familiar with the format of multiple-choice, completion and short-answer items. Students should be encouraged to make an attempt at every item on the assessment. There is no penalty for guessing on multiple-choice items. Partial credit can be earned on short-answer items. Teachers can find many new sample items for student practice in the *Mathematics Assessment Updates for 2012* document that can be accessed at: http://www.k12.wa.us/Mathematics/Resources.aspx

This document can be accessed electronically at:
http://www.k12.wa.us/Mathematics/LessonsLearned.aspx

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General Considerations on all EOC Exams and Retake Exams

To improve scores on EOC exams, students should:

Use exact values given in the prompt without rounding or approximating.	Examples: -When 14.7^{o} is given as an angle measure, students should not use 15^{o}. - When $\frac{2}{3}$ is given as a coefficient in an equation, students should not use decimal approximations, 0.6, 0.7, 0.66, 0.67, 0.667, etc., when solving the equation.
Round or truncate only the final answer. Students should not "round as they go."	When asked to provide an answer to a specific level of precision, like "to the nearest tenth," students should not round or truncate intermediate calculations. Rounding or truncating "as you go" may result in an answer that is not within an acceptable range of answers.
Round final answers to the appropriate place value when specified.	Example: A new $20,000 car depreciates at a rate of 8% per year. Find the value of the car, to the nearest dollar, after 5 years. Solution: $20{,}000(1-0.08)^5$ = 13,181.6304…. Correct answer: $13,182
When approximating the value of an expression involving a radical or trigonometric expression, round to the nearest hundredth.	Rounding inappropriately may result in an answer that is not within an acceptable range of answers.
Use unique variables to represent different unknowns when writing and/or using equations or expressions to solve problems.	Be sure to use different variables to represent different measures. Using the same variable to represent different measures can confound scoring.
Include the attribute being measured and appropriate units when defining variables.	The definition "d=driving" is not clear whether time or distance is being measured. The definitions "d=time driving in hours" or "d=distance driving in miles" are clear because they include both the attribute and the unit.
Use precise vocabulary when providing a written verbal description.	Avoid using "it" in place of specific mathematical terms in verbal descriptions or in comparisons. Examples: - For the line y = 2x-5, "it is positive" is not a specific description. Instead, the student could say "the slope is positive" or "the y-intercept is negative". - When asked to compare two values, "it is larger" is not a specific comparison. Instead, "value A is larger than value B" is a clear comparison.
Show work to support an answer when specified.	Students who do not show work when required will not earn full credit.

Clearly label any calculations and/or work when using a diagram to solve a problem.	Student work that is not clear to the scorer will not earn full credit.
Avoid run-on equations when showing work.	No credit is given for run-on equations when scoring student work. Example: Solve the equation $2x+8=18$. -Acceptable: $2x=18-8=10 \rightarrow x=\frac{10}{2}=5$ -Acceptable: $x=\frac{18-8}{2}=5$ -Run-on (unacceptable): $x=18-8=10\div 2=5$
Use specific information from the prompt, diagram, equation, graph or student work when asked to provide an example to support a statement or conclusion.	Students often are too vague in the support of their conclusions; "because those are the numbers in the equation" or "because that's what's in the graph" do not use specific information. Students who do not provide support using specific information when asked to do so will not earn full credit.
Use a particular strategy or procedure when specified.	For example, if a student is asked to show how they solve an equation, "guess and check" does not show how to solve the equation and will not earn full credit.
Determine when a label is required for a short-answer item.	The question: **"What is the height of the tree? _____"** does require the student include a label to indicate the unit of measure. Students do not need to include labels when they are included in the question. When answering the following questions: **"What is the height of the tree to the nearest foot?"** or **"What is the height of the tree? _____ feet"** students do not need to provide additional labels because the unit of measure is supplied in the item.
Avoid giving a "yes/no" answer to a non-yes/no question.	When answering the following question: **"Is the statement valid or invalid?"** The student should write "valid" or "invalid," not "yes" or "no".
Be familiar with the Mathematics Formula Sheets for End-of-Course Exams.	The Mathematics Formula Sheets for End-of-Course Exams are available in the Test and Item Specifications at http://www.k12.wa.us/Mathematics/TestItemSpec.aspx. The formula sheets are not secure and teachers are encouraged to familiarize students with the format and content of the formula sheets prior to testing.
Be familiar with vocabulary used in EOC exam items.	A link to the list of EOC vocabulary terms is: http://www.k12.wa.us/Mathematics/TestItemSpec.aspx

Considerations for Year 1 Exams: Algebra 1, Integrated Math 1, Year 1 Retake

To improve scores on Year 1 exams, students should:

A1.1.E

Be familiar with both exponential growth and decay.	Students should be familiar with contexts that involve exponential decay, not just exponential growth.
Use an exponential model to solve problems that can be modeled by exponential functions.	Students should write and use an exponential model when appropriate. Students who choose to use a table to solve these types of problems should not round answers along the way.

A1.3/M1.2

Be familiar with and understand function notation, *f(x).*	Students should be familiar with functions written with function notation, should know how to evaluate *f(a)*, and should know how to solve equations of the form *f(x)=b*.
Use precise vocabulary when given a written verbal description involving functions.	When verbally describing the expression *f*(*x*): -Correct: *f* of *x* -Incorrect: *f* multiplied by *x* Use function terminology: relation, element, domain, range

A1.3.A/M1.2.A

Be able to describe the range or domain of a function verbally or symbolically.	When asked to identify the domain and/or range of a function, students are expected to be able to do so either verbally or symbolically. Example: "all real numbers greater than 3" or "all real numbers x>3"
If a student chooses to use set or interval notation, he/she should use set or interval notation correctly.	For example, if describing "all real numbers greater than 3: -Correct Use Set notation: $\{ x \in \mathbb{R} \mid x > 3 \}$ Interval notation: $(3, \infty)$ -Incorrect Use $\{ x > 3 \}$ $(3, \infty]$

A1.3.B

Be able to work with functions represented algebraically, graphically, numerically or verbally.	Students should be able to work with a function represented by an equation, a graph, a table of values, or a verbal description. Example: Function A is defined by the data table shown. Determine *f*(12).

x	*f*(*x*)
4	$17.25
6	$20.75
8	$24.25

A1.4.C/M1.3.C

Represent an intercept as a single value or an ordered pair.	Students should represent the intercept in the form specified in the item. When students use an ordered pair, the order must be correct to represent the intercept; (0, 4) represents a *y*-intercept while (4, 0) represents an *x*-intercept.
Represent slope as a single numerical value.	Slope should be a numerical value, not a description of movement. Slope does not include a variable; in the slope-intercept form ($y = mx + b$), only the value m is the slope. For example, when identifying the slope of the graph of the equation $y = -\frac{1}{2}x + 3$: Correct: $-\frac{1}{2}$ Incorrect: "down 1, over 2" Incorrect: $-\frac{1}{2}x$
Understand that slope represents a ratio between two changing values.	When describing the slope, students should make reference to both changing values; i.e., "miles per hour" or "revolutions per second." When asked to interpret the slope, "rise over run" is not an interpretation.
Use precise language when describing graph characteristics.	Students should explain which part of the graph is positive or negative (ie. the slope, the y-intercept). The phrase "the graph is negative (or positive)" has no mathematical meaning.

A1.4.E

Understand how changes in parameters cause a reflection of a function's graph.	Students should know the differences between a reflection over the x-axis and a reflection over the y-axis. A change in the parameter of a function will not create a rotation of the graph.

A1.5.C

Be able to factor a quadratic expression with a leading coefficient other than 1.	Students should be able to factor a quadratic expression $ax^2 + bx + c$ where $a \neq 1$. Example: $2x^2 + 3x - 5 = (2x + 5)(x - 1)$

A1.6.A/ M1.5.A

Use comparative language when comparing data sets.	Examples: "greater than," "less than" or "equal to." Phrases such as "about the same" or "close together" do not give enough information about how the data sets compare.
Use statistical language to describe data sets.	Statistical language includes maximum, minimum, mean, median, mode, range, interquartile range, etc. The term "average" is not specific enough because it can be associated with multiple measures of center—mean, median and mode.

A1.6.D/M1.3.F

Understand that values that satisfy an equation can be used to approximate data values in the set but may not be actual data values in the set.	For example, when the equation $y = 0.08x - 3.2$ is used to model a set of data comparing student height in centimeters to their shoe size, there may not be a student in the set who is exactly 152.5 centimeters tall and has a shoe size of 9. Additionally, the equation should not be used to infer that for every 12.5 centimeters of height the shoe size increases by one. The equation may show a trend of increase or average increase but does not show exact increases between data values.
Understand that slope represents a ratio between two changing values.	When describing slope, students should make reference to both changing values; i.e., "miles per hour" or "feet per second."
When asked to draw a trend line that fits the data, students should draw a single line that comes closest to connecting the data points on a scatterplot.	Students should not draw multiple line segments connecting each point. A trend line does not necessarily begin at the origin (0, 0).

A1.6.E

Understand the difference between strong and weak correlation.	Data with strong correlation has a more evident systematic relationship than data with weak correlation. Strong correlation does not mean steeper; weak correlation does not mean less steep.

A1.7.C

Be familiar with and understand notation of sequences defined recursively.	Given a recursive formula and one term in a sequence, students should be able to find another term in the sequence. Students should be familiar with recursive formulas that define the nth term or the (n+1) term. Examples: $a_n = a_{n-1} + 2$ or $a_{n+1} = 2 \cdot a_n$
Understand the differences between arithmetic and geometric sequences.	The terms "arithmetic" and "geometric" are used in assessment items, and students are expected to know the meanings of these terms.

Considerations for Year 2 Exams: Geometry, Integrated Math 2, Year 2 Retake

To improve scores on Year 2 EOC exams, students should:

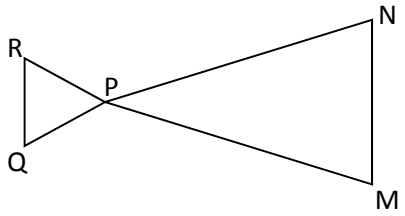
G.1.D/M2.3.C

Become fluent writing the inverse, converse and contrapositive of a valid statement.	Students should use "if" and "then" when writing the inverse, converse and contrapositive of a valid statement.
When writing the inverse or contrapositive of a statement, students need to take care when writing equivalents for "not" statements.	Examples: -"A number is even" is equivalent to "A number is not odd." -"A number is positive" is not equivalent to "A number is not negative" because the number zero is neither positive nor negative. -"A figure is not a quadrilateral" is not equivalent to "a figure has more than four sides" because a triangle is not a quadrilateral, but it does not have more than four sides.
Use the terms valid/invalid appropriately.	The terms true/false and valid/invalid are not interchangeable. A statement can be true in some cases and false in others; a statement is not valid in some cases and invalid in others. Valid means true in all cases.
Be able to provide a counterexample for an invalid conditional statement.	A counterexample for the conditional statement "If *p*, then *q*" must provide an example where *p* is true but *q* is false.

G.2.A/M1.4.E

Be able to determine the slope of a line perpendicular to a given line.	Students should be able to determine the slope of a line perpendicular to a given line from a graph or equation in standard or slope-intercept form.
Represent slope as a single numerical value.	Slope should be a numerical value, not a description of movement. Slope does not include a variable; in the slope-intercept form ($y = mx + b$), only the value m is the slope. For example, when identifying the slope of the graph of the equation $y = -\frac{1}{2}x + 3$: Correct: $-\frac{1}{2}$ Incorrect: "down 1, over 2" Incorrect: $-\frac{1}{2}x$

G.2.B /M1.4.F

Uniquely identify angles.	In the figure, $\angle M$ is specific enough to uniquely identify one angle; however, $\angle P$ is not specific enough to identify one angle. Using $\angle NPM$ is specific enough to identify one angle. 
Accurately name types of angles created by lines cut by a transversal.	Students should know the following types of angles: - Alternate interior - Alternate exterior - Consecutive - Corresponding
Accurately describe congruent angles created by parallel lines cut by a transversal.	Examples: 2 non-parallel lines cut by a transversal have non-congruent corresponding angles; 2 parallel lines cut by transversal have congruent corresponding angles

G.3.B/M2.3.F

State all given information when constructing a proof.	A student who fails to state all given information within his/her proof may not earn full credit.
Use specific information from diagrams or the prompt when constructing or completing proofs.	Examples: -Use multiple points to uniquely describe sides and angles. -Use correct order in labeling to identify corresponding sides, angles and vertices.
Avoid using non-standard acronyms; not all acronyms are known by the scorer.	Examples: -Acceptable acronyms: ASA, SSS -Avoid non-standard acronyms: AEA for "alternate exterior angles"
Use specific names or descriptions of properties/theorems/postulates when constructing or completing proofs.	Examples: -"Parallel Lines Property" or "Parallel Line Laws" are not specific or sufficient support. -"Theorem 4.1" does not adequately describe the theorem used.
Avoid using CPCTC to justify triangle congruence.	Corresponding Parts of Congruent Triangles are Congruent (CPCTC) is not valid justification to establish triangle congruence.

G.3.D/M2.3.G

When approximating the value of an expression involving a radical, round to the nearest hundredth.	Students should use at least two decimal places when approximating square roots as specified on page 2 of the Mathematics Formula Sheets for End-of-Course Exams.

G.3.E/M2.3.H

Be familiar with and have access to a calculator that includes trigonometric ratio functionality during Year 2 EOC exams or a table of trigonometric values.	Access the Calculator Policy at: http://www.k12.wa.us/Mathematics/CalculatorPolicy.aspx An end-of-course trigonometric table is available at http://www.k12.wa.us/TestAdministration/FormsReports/.
Make sure the calculator is in "degree" mode when calculating trigonometric ratios.	Student calculators should be set to degree mode, not radian mode, prior to the EOC exam.
Use precise vocabulary when providing a written description.	Example: When describing the expression $\tan(72^\circ)$: -Correct: the tangent of 72 degrees -Incorrect: the tangent multiplied by 72 degrees
Understand and use proper notation involving inverse trigonometric functions.	Example: - $\tan^{-1}(1)$ refers to an angle with a tangent equal to 1. - $\tan^{-1}(1)$ is not equivalent to $(\tan(1))^{-1}$ or to $1/\tan(1)$.
Use exact values given in the prompt without rounding or approximating.	Examples: -When 14.7^o is given as an angle measure, students should not use 15^o. - When $\frac{2}{3}$ is given as a coefficient in an equation, students should not use decimal approximations, 0.6, 0.7, 0.66, 0.67, 0.667, etc., when solving the equation.
Round or truncate only the final answer. Students should not "round as they go."	When asked to provide an answer to a specific level of precision, like "to the nearest tenth," students should not round or truncate intermediate calculations. Rounding or truncating "as you go" may result in an answer that is not within an acceptable range of answers.
When approximating the value of a trigonometric expression, round to the nearest hundredth.	Students should use at least two decimal places when approximating trigonometric ratios as specified on page 2 of the Mathematics Formula Sheets for End-of-Course Exams.
Know how to solve a trigonometric equation when x is in the denominator of an expression.	Example: Solve the equation $\sin(35^\circ) = \frac{27}{x}$ Solution: $x = \frac{27}{\sin(35^\circ)} = 47.073...$
Know how to solve problems involving angles of elevation without a given diagram.	Students perform fairly well when given a labeled diagram; however, students struggle when given a written description of a situation without a diagram. Students should be able to use an angle of elevation to find a missing side without being given a diagram. Students should also be able to find an angle of elevation when given two side lengths in context without being given a diagram.

G.4

Use the variables *x* and *y* when writing equations for lines or circles in the coordinate plane.	In short-answer items, students may use variables other than *x* and *y*, but those variables must be defined, either within the item or by the student.
Assume an angle is a right angle only if it is identified with an angle measure or appropriate right angle mark.	Example: Only one angle in the figure below is guaranteed to be a right angle. 

G.4.A/M1.3.H

Be able to determine the slope of a line perpendicular to a given line through a specified point.	Students should be able to determine the equation of a line through a specified point that is perpendicular to a given line from a graph or equation in standard or slope-intercept form.
Be familiar with linear functions whose equations are given in standard form.	Students should be able to manipulate equations and determine the slope of a line whose equation is given in standard form.
Be familiar with equations for vertical and horizontal lines.	Students should be able to identify the equation of a horizontal or vertical line through a specified point.

G.4.B/M2.3.L

Use graph paper to graph accurately.	When solving a problem graphically, students should use graph paper to make accurate graphs.
Know the locations of each quadrant.	Know the location of each quadrant on the coordinate axes. Students should understand that points located on the x- or y-axis are not in a quadrant.

G.4.C/M2.3.M

Use properties of triangles or quadrilaterals to solve problems.	When given a figure in the coordinate plane with coordinates labeled, students should not “eyeball” their answers. Students should verify their answers using the appropriate properties and values from the graph.

G.4.D

When given the equation of a circle and a line that intersect, learn how to algebraically solve for the point(s) of intersection.	When given the equations of two functions that intersect, students should know how to algebraically solve the system of equations by substitution.

G.5.A

Be familiar with transformation rules presented in the form $(x, y) \rightarrow (ax + h, by + k)$	For example, the transformation $(x, y) \rightarrow (x - 7, y - 3)$ means that all points will move 7 units to the left and 3 units down.

G.5.D

Understand that rotations include both a magnitude and direction.	When describing a rotation, students should give both the number of degrees and the direction.
Know clockwise vs. counterclockwise when describing a rotation.	When describing a rotation other than 180 degrees, students should specify the direction of rotation as well as the number of degrees.

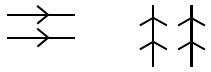
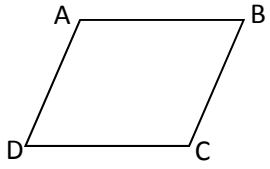
G.6.A

Be sure to find the area of the appropriate sector.	When a circle is divided into two sectors, be sure to find the area specified in the problem.

G.6.E

Know how tolerances and margins of error relate to the range of acceptable values in a given situation.	For example, if a tolerance requires staying within ¼ inch of 3 inches, the acceptable range of values is from 2.75 inches to 3.25 inches.

G.7.E

Know how to label parallel lines in a geometric diagram.	Use the following symbols to label parallel lines: 
Use a straight edge to draw accurate diagrams.	
When asked to draw a geometric figure, label all given vertices.	For example, if asked to draw parallelogram ABCD, label vertices appropriately. Correct:  Incorrect: