

2011 Lessons Learned from Scoring Student Work On Grades 6-8 MSP

Rangefinding, scoring and data review of items on the Mathematics Measurements of Student Progress (MSP) provides the opportunity to see hundreds of student responses at each grade and/or course level and to evaluate data summarizing student performance. The Mathematics Assessment Team would like to share observations about student responses and areas of mathematics where students appear to be struggling. Lessons Learned from Scoring Student Work lists actions students could take to increase their scores on the state assessments. Because new standards are being assessed, we have also added descriptions of content that was particularly difficult for students. Grades and/or courses will be listed separately.

In general, students fail to earn points toward a better score because of incomplete responses or incomplete mathematical representations. Students could improve their scores by:

- answering the question or completing each task in the prompt;
- using bullets as a checklist to make sure the response is complete;
- checking to see that they transcribe the correct numbers in the prompt and checking their computations for accuracy.

Information from Lessons Learned should be modeled, practiced and used throughout the school year. Students should be familiar with the format of multiple-choice, completion and short-answer items. Students should be encouraged to make an attempt at every item on the assessment. There is no penalty for guessing on multiple-choice items. Partial credit can be earned on short-answer items. Teachers can find many new sample items for student practice in the *Mathematics Assessment Updates for 2012* document that can be accessed at: <http://www.k12.wa.us/Mathematics/Resources.aspx>

Students taking the assessments online will benefit from instruction and practice on the use of online tools provided on the demo and tutorial. The demo and tutorial will be available in January 2012. Visit the online testing website for more information about online testing in Washington: <http://www.k12.wa.us/assessment/StateTesting/OnlineTesting.aspx>

This document can be accessed electronically at:
<http://www.k12.wa.us/Mathematics/LessonsLearned.aspx>

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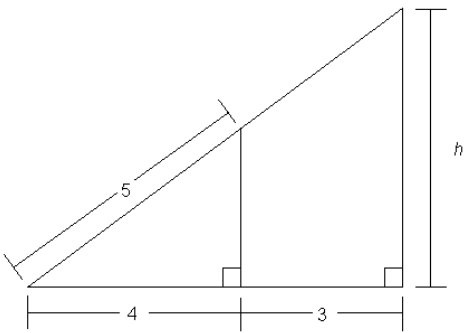
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General Considerations on Grades 6-8 MSP

To improve scores on the MSP, students should:

<p>Use correct notation when using exponents to label area and volume measures.</p>	<p>Examples:</p> <ul style="list-style-type: none"> - Correct notations using exponents for area and volume measures include: <ul style="list-style-type: none"> • 20 feet² • 20 ft² • 20 feet³ • 20 ft³ - Incorrect notations using exponents for area and volume measures include: <ul style="list-style-type: none"> • 20² feet (this answer is equivalent to “400 feet”) • 20¹² (using ' to indicate feet, but appears to be “20 to the 12th power”) • 20²¹ (using ' to indicate feet, but appears to be “20 to the 21st power”) • 20³ feet (this answer is equivalent to “8,000 feet”) • 20¹³ (using ' to indicate feet, but appears to be “20 to the 13th power”) • 20³¹ (using ' to indicate feet, but appears to be “20 to the 31st power”) 								
<p>Use exact values given in the prompt without rounding or approximating.</p>	<p>Examples:</p> <ul style="list-style-type: none"> - When 14.7° is given as a temperature, students should not use 15°. - When $\frac{2}{3}$ is given in an expression or equation, students should not use decimal approximations, 0.6, 0.7, 0.66, 0.67, 0.667, etc., when evaluating the expression or solving the equation. 								
<p>Round or truncate only the final answer.</p>	<p>In a multi-step problem, students should not round or truncate intermediate calculations. Rounding or truncating “as you go” may result in an answer that is not within an acceptable range of answers.</p>								
<p>Include all the numbers being operated on when describing mathematical operations.</p>	<p>Students sometimes make incomplete statements by not including all the numbers that were used in an operation. Without knowing which numbers were used, the work is not clear and therefore does not receive credit.</p> <table border="1" data-bbox="472 1247 1430 1440"> <thead> <tr> <th data-bbox="472 1247 951 1285">Incomplete Statement</th> <th data-bbox="951 1247 1430 1285">Complete Statement</th> </tr> </thead> <tbody> <tr> <td data-bbox="472 1285 951 1356">I added the numbers to 6 and got 18.4</td> <td data-bbox="951 1285 1430 1356">I added 7.8 and 4.6 to 6 and got 18.4</td> </tr> <tr> <td data-bbox="472 1356 951 1394">I multiplied them together</td> <td data-bbox="951 1356 1430 1394">I multiplied 3.5 and 2.7 together</td> </tr> <tr> <td data-bbox="472 1394 951 1440">I divided by 3 to get my answer</td> <td data-bbox="951 1394 1430 1440">I divided $\frac{2}{5}$ by 3 to get my answer</td> </tr> </tbody> </table>	Incomplete Statement	Complete Statement	I added the numbers to 6 and got 18.4	I added 7.8 and 4.6 to 6 and got 18.4	I multiplied them together	I multiplied 3.5 and 2.7 together	I divided by 3 to get my answer	I divided $\frac{2}{5}$ by 3 to get my answer
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<p>Correctly describe subtraction and division operations.</p>	<p>Students reverse the order of the numbers when describing subtraction and division, and write “I subtracted fifteen from three” when they are trying to describe $15 - 3$ or “I divided three by fifteen” when trying to describe $3 \overline{)15}$.</p> <p>- Sample acceptable descriptions for $15 - 3$:</p> <ul style="list-style-type: none"> • fifteen minus three • take three away from fifteen • subtracted three from fifteen <p>- Sample acceptable descriptions for $15 \div 3$ or $3 \overline{)15}$:</p> <ul style="list-style-type: none"> • fifteen divided by three • three divided into fifteen • divided three into fifteen • three goes into fifteen
<p>Use unique variables to represent different unknowns when writing and/or using equations or expressions to solve problems.</p>	<p>Using the same variable to represent different unknowns can make student responses unclear and confound scoring.</p> <p>- 8th grade Example:</p>  <p>When asked to determine the length of side h of the triangle, students sometimes write these two equations: $4^2 + h^2 = 5^2$ and $\frac{4}{7} = \frac{3}{h}$. Using h in both equations is confusing as the h in the first equation is the height of the smaller triangle and the h in the second equation is the height of the larger triangle.</p> <p>Rather, students should use different variables and define the variables.</p>
<p>Include the attribute being measured and appropriate units when defining variables.</p>	<p>The definition “d=driving” is not clear whether time or distance is being measured. The definitions “d=time driving in hours” or “d=distance driving in miles” are clear because they include both the attribute, “time” or “distance,” and the unit, “hours” or “miles.”</p>

<p>Use specific information from the prompt, diagram, equation, graph, etc. when supporting a statement or conclusion.</p>	<p>Students often are too vague in the support of their conclusions; “because those are the numbers in the equation” or “because that’s what’s in the graph” do not use specific information.</p> <p>- Sample specific information:</p> <ul style="list-style-type: none"> • “because the temperature in July was 76 degrees” • “because you are adding 110.45 in the equation” • “because the graph shows Deshawn had 19 more marbles than Jonah” • “because the distance between the tower and the wall is 5 more feet than the distance between the wall and the sidewalk.”
<p>Use equal signs correctly when writing equations.</p>	<p>If a student adds 14.3 and 7.05 and then multiplies the sum by 12, they sometimes write a run-on equation, $14.3+7.05 = 21.35 \times 12 = 256.2$, to show their work. This run-on equation is mathematically incorrect because all three expressions are not equal to each other. Run-on equations do not receive credit for showing work. Rather, students can do things such as:</p> <ul style="list-style-type: none"> • show different steps on different lines $14.3+7.05 = 21.35$ $21.35 \times 12 = 256.2$ • verbally describe the steps they took “First I added 14.3 and 7.05 and got 21.35. Then I multiplied 21.35 and 12 and got 256.2.” • use order of operations and equivalent expressions $(14.3+7.05) \times 12 = 21.35 \times 12 = 256.2$ <p>If students do use the order of operations and equivalent expressions, it is important to represent the order of operations correctly, using parentheses when needed.</p>
<p>Use labels to describe equivalencies and when solving problems with scales, rates, or similar figures.</p>	<p>When describing measurement conversions, it is acceptable for a student to write “1 foot = 12 inches;” however, students often leave off the labels and write “1 = 12” which is not a correct statement. The same is true when students are solving problems with scales, rates, or similar figures. It is acceptable to write “1 inch on the model = 15 feet of the wall” or even “1 inch = 15 feet,” but “1 = 15” is not correct and not acceptable.</p>

<p>Write equations using only numbers, appropriate mathematical symbols, and variables.</p>	<p>Students often include words in equations, which is not acceptable. Students should have practice using mathematical symbols for operations (e.g., +, -, •, ÷) and variables in place of the words.</p> <p>Examples:</p> <table border="1" data-bbox="472 359 1419 543"> <thead> <tr> <th>Not acceptable</th> <th>Acceptable</th> </tr> </thead> <tbody> <tr> <td>3 times 4 equals 12</td> <td>$3 \cdot 4 = 12$</td> </tr> <tr> <td>The number of apples + 6 = 8</td> <td>$a + 6 = 8$</td> </tr> <tr> <td>The speed she is driving multiplied by the times she is driving equals the distance she drove.</td> <td>$rt = d$</td> </tr> </tbody> </table> <p>Additionally, students sometimes use inappropriate symbols when writing equations, such as writing “3:8 = 6:16” to represent the proportion $\frac{3}{8} = \frac{6}{16}$ or “d : 45t+9” instead of the equation $d = 45t + 9$.</p>	Not acceptable	Acceptable	3 times 4 equals 12	$3 \cdot 4 = 12$	The number of apples + 6 = 8	$a + 6 = 8$	The speed she is driving multiplied by the times she is driving equals the distance she drove.	$rt = d$						
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<p>Use parentheses appropriately when writing equations and expressions.</p>	<p>Students often omit parentheses, resulting in an equation or expression that is awkward or incorrect based on the order of operations.</p> <table border="1" data-bbox="472 835 1408 995"> <thead> <tr> <th>Correct Use of Parentheses</th> <th>Awkward without Parentheses</th> </tr> </thead> <tbody> <tr> <td>$a = 3t - (-4)$</td> <td>$a = 3t - -4$</td> </tr> <tr> <td>$t \times (-4) = -11 \times (-8)$</td> <td>$t \times -4 = -11 \times -8$</td> </tr> <tr> <td>$3 \times (-5) \div (-2)$</td> <td>$3 \times -5 \div -2$</td> </tr> </tbody> </table> <table border="1" data-bbox="472 1033 1408 1152"> <thead> <tr> <th>Correct Use of Parentheses</th> <th>Incorrect without Parentheses</th> </tr> </thead> <tbody> <tr> <td>$(6 + 4) \times 3 = 30$</td> <td>$6 + 4 \times 3 = 30$</td> </tr> <tr> <td>$15 \div (3 + 2) = 3$</td> <td>$15 \div 3 + 2 = 3$</td> </tr> </tbody> </table>	Correct Use of Parentheses	Awkward without Parentheses	$a = 3t - (-4)$	$a = 3t - -4$	$t \times (-4) = -11 \times (-8)$	$t \times -4 = -11 \times -8$	$3 \times (-5) \div (-2)$	$3 \times -5 \div -2$	Correct Use of Parentheses	Incorrect without Parentheses	$(6 + 4) \times 3 = 30$	$6 + 4 \times 3 = 30$	$15 \div (3 + 2) = 3$	$15 \div 3 + 2 = 3$
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<p>Use appropriate approximations for π (pi) when calculating with π.</p>	<p>Appropriate approximations for π include 3.14, 3.14159, $\frac{22}{7}$, or, for grade 7 and grade 8, the value given by the π button on a calculator. Students should be given practice deciding which approximation is most appropriate for a given problem.</p> <p>Students may give exact answers in terms of π, such as 14π.</p> <p>Three (3) and 3.1 are not appropriate approximations for π.</p>														

<p>Understand the value of a numeric expression is a single number.</p>	<p>When students are asked to determine the value of a numeric expression, students sometimes give another, equivalent expression as an answer. However, those expressions do not show the value and do not earn credit.</p> <table border="1" data-bbox="472 331 1419 579"> <thead> <tr> <th>Original Expression</th> <th>Equivalent Expression (not the value)</th> <th>Value of the expression</th> </tr> </thead> <tbody> <tr> <td>$3 + 4.5 \times 2.6$</td> <td>$3 + 11.7$</td> <td>14.7</td> </tr> <tr> <td>$7 - 10 \times 2$</td> <td>-13</td> <td>13</td> </tr> <tr> <td>$\frac{3^2 + (7 - 2)^3}{(2^3)^2}$</td> <td>$\frac{3^2 + 5^3}{2^6}$</td> <td>$\frac{134}{64}$ or $\frac{67}{32}$ or $2\frac{3}{32}$</td> </tr> </tbody> </table>	Original Expression	Equivalent Expression (not the value)	Value of the expression	$3 + 4.5 \times 2.6$	$3 + 11.7$	14.7	$ 7 - 10 \times 2 $	$ -13 $	13	$\frac{3^2 + (7 - 2)^3}{(2^3)^2}$	$\frac{3^2 + 5^3}{2^6}$	$\frac{134}{64}$ or $\frac{67}{32}$ or $2\frac{3}{32}$
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<p>Use “about,” “approximately,” etc. and the symbol “\approx” appropriately.</p>	<p>When determining the area or circumference of circles, the volume or surface area of cylinders, the probability of certain events, and some triangle side lengths using the Pythagorean Theorem, it is appropriate for a student to state the answer is “about 10 square inches,” “approximately 75.4 cubic feet,” “close to 34%,” or “\approx 14 inches.” However, students also use these terms and symbol at times when it is inappropriate. For example:</p> <p>Example 1:</p> <ul style="list-style-type: none"> - A bookshelf has 7 shelves. The mean number of books on each shelf is 4. Six of the shelves have these number of books: <p style="text-align: center;">2, 2, 5, 6, 3, 2</p> <p>How many books are on the seventh shelf? Students incorrectly say “about 8,” when it is exactly 8.</p> <p>Example 2:</p> <ul style="list-style-type: none"> - Mario is making decorations using markers and construction paper. He has 3 different markers, one red, one green, and one purple. The construction paper comes in 5 different colors. Mario will use one marker and one piece of construction paper to make each decoration. <p>How many different decorations can Mario make using one marker and one piece of construction paper? Students incorrectly say “about 15,” when it is exactly 15</p> <p>Example 3:</p> <ul style="list-style-type: none"> - Solve the equation for x. <p style="text-align: center;">$4x + 3 = 11$</p> <p>What is the value of x? Students incorrectly write $x \approx 2$ when $x = 2$.</p>												

Check the accuracy of transcribed work done on scratch or graph paper.	Students will need to do work on scratch and/or graph paper while taking the online MSP. Students must take care that the work they are typing in the computer matches the work they have done on the scratch or graph paper. Students should use the online tutorial to practice strategies to check that what they have typed correctly represents their work.
Be familiar with vocabulary used in MSP items.	A link to the list of MSP vocabulary terms is: http://www.k12.wa.us/Mathematics/TestItemSpec.aspx

Considerations for Grade 6:

To improve scores on the MSP, students should:

6.1.H

<p>Represent operations with fractions correctly.</p>	<p>When solving problems involving fractions, students sometimes incorrectly represent finding equivalent fractions. For example, when finding an equivalent fraction for $\frac{3}{5}$, students will sometimes write $\frac{3}{5} \times 8 = \frac{24}{40}$ which is not a correct equation. Students should show that both the numerator and denominator are being multiplied by the same factor ($\frac{3}{5} \times \frac{8}{8} = \frac{24}{40}$) or just write the equivalency without the multiplication ($\frac{3}{5} = \frac{24}{40}$)</p>
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6.2.A

<p>Have more practice writing equations to represent information in a table.</p>	<p>Students sometimes take the values in the table and connect them incorrectly to the corresponding variable. For example, when writing an equation to represent the relationship between time (t) and distance (d) in the table below, students sometimes write the equation “$0.5t = 120d$” which does not represent the data in the table.</p> <table border="1" data-bbox="467 877 1239 955"> <tr> <td>Time (hours)</td> <td>0.5</td> <td>1.0</td> <td>1.5</td> <td>2.0</td> </tr> <tr> <td>Distance (feet)</td> <td>120</td> <td>240</td> <td>360</td> <td>480</td> </tr> </table> <p>Students can check the equation they write to verify that it represents the data in the table by substituting different values from the table for the variables.</p>	Time (hours)	0.5	1.0	1.5	2.0	Distance (feet)	120	240	360	480
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Distance (feet)	120	240	360	480							
<p>Understand that writing an equation to represent information in a table is different than describing patterns in a table.</p>	<p>Students sometimes describe patterns in the table instead of writing an equation. For example, when asked to write an equation to represent the relationship between time (t) and distance (d) in the table below, students sometimes write “the time is adding 0.5 and the distance is going up by 120 each time.”</p> <table border="1" data-bbox="467 1276 1239 1354"> <tr> <td>Time (hours)</td> <td>0.5</td> <td>1.0</td> <td>1.5</td> <td>2.0</td> </tr> <tr> <td>Distance (feet)</td> <td>120</td> <td>240</td> <td>360</td> <td>480</td> </tr> </table>	Time (hours)	0.5	1.0	1.5	2.0	Distance (feet)	120	240	360	480
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6.3.F, G

<p>Understand one or more ways to represent probability.</p>	<p>There are many ways students can represent probability as a number between 0 and 1, inclusive, including decimals (0.25), percents (25%), and fractions ($\frac{1}{4}$).</p> <p>Verbal descriptions, such as “1 in 4,” “1 out of 4,” “1 of 4,” or “1 in 4 chance,” are also appropriate ways to represent probability.</p> <p>However, students sometimes represent probability using ratios, such as “1:4” or “1:3.” Ratios are not appropriate ways to represent probability because ratios do not represent a number between 0 and 1, inclusive.</p>
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6.4.B

Develop strategies to decompose composite figures into rectangles, triangles, and parts of circles.	Often composite figures can be broken down into a variety of simpler figures. However, depending on the given information, certain decompositions make it easier to determine the area or perimeter of the composite figure. This is especially true when measurements necessary for determining area or perimeter must be derived from the given information.
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6.4.C

Have more practice determining measurements of a circle.	Students are better able to determine the circumference of a circle given the radius or diameter than determine the radius, diameter, or area of a circle given the circumference.
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6.4.E, F

Fully develop their understanding of the concept of surface area.	<p>Students need to understand that surface area is the sum of the areas of all the faces of a prism or pyramid. They often confuse surface area with volume, especially when working with rectangular prisms.</p> <p>Students particularly struggle with determining the surface area of pyramids, perhaps because there is no one formula for determining the surface area. Students should have a variety of experiences determining the surface area of pyramids with different bases. Teachers can help reinforce the concept of surface area by helping students understand how the area of each face is determined by relating the three-dimensional pyramid to its two-dimensional net.</p>
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Considerations for Grade 7:

To improve scores on the MSP, students should:

7.1.C

<p>Develop fluency and accuracy of computations with rational numbers.</p>	<p>Students need practice doing computations with negative numbers, especially subtraction such as $-7.5 - 3.7$, and computations that result in negative values, such as $16\frac{1}{4} - 23\frac{1}{2}$.</p> <p>Students need practice completing computations that include combinations of fractions and decimals, such as $5.375 + 18\frac{4}{5}$ or $0.25 \cdot (-\frac{4}{7})$. This practice should include decisions about which form of the numbers will result in an exact, correct answer. For example, in $5.375 + 18\frac{4}{5}$, a student could either convert the decimal to a fraction or the fraction to a decimal to get to an exact answer. However, in $0.25 \cdot (-\frac{4}{7})$, it is better that students convert the decimal to a fraction; converting $-\frac{4}{7}$ to a decimal is unlikely to result in an exact, correct answer.</p>
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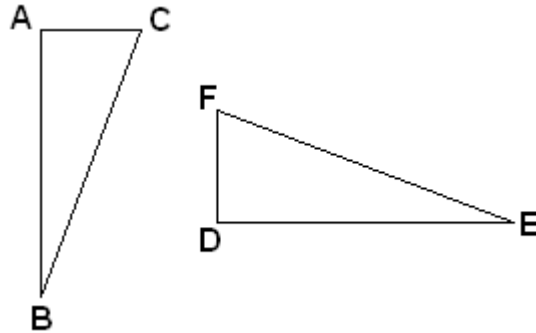
7.2.B

<p>Correctly represent the steps used to solve a proportion.</p>	<p>Students do a good job of setting up proportions; however, students sometimes incorrectly represent the steps to solve the proportion.</p> <table border="1" data-bbox="597 974 1304 1465"> <thead> <tr> <th data-bbox="597 974 954 1010">Correct Representations</th> <th data-bbox="963 974 1304 1010">Incorrect Representations</th> </tr> </thead> <tbody> <tr> <td data-bbox="597 1016 954 1230"> $\frac{x}{7} = \frac{27}{21}$ $21x = 189$ $x = 9$ </td> <td data-bbox="963 1016 1304 1230"> $\frac{x}{7} = \frac{27}{21} = 21x = 7 \times 27$ </td> </tr> <tr> <td data-bbox="597 1236 954 1310"> $\frac{x}{7} = \frac{27}{21}, 21x = 189, x = 9$ </td> <td data-bbox="963 1236 1304 1310"> $\frac{x}{7} = \frac{27}{21}, 21x = 189 = 9$ </td> </tr> <tr> <td data-bbox="597 1316 954 1390"> $\frac{x}{7} = \frac{27}{21}, x = 9$ </td> <td data-bbox="963 1316 1304 1390"> $\frac{x}{7} = \frac{27}{21} = 9$ </td> </tr> <tr> <td data-bbox="597 1396 954 1465"> $\frac{x}{7} = \frac{27}{21} \text{ so } x = 9$ </td> <td data-bbox="963 1396 1304 1465"> $\frac{x}{7} = \frac{27}{21} = x = 9$ </td> </tr> </tbody> </table> <p>The incorrect representations are incorrect mathematical equations as the expressions are not all equal to each other.</p>	Correct Representations	Incorrect Representations	$\frac{x}{7} = \frac{27}{21}$ $21x = 189$ $x = 9$	$\frac{x}{7} = \frac{27}{21} = 21x = 7 \times 27$	$\frac{x}{7} = \frac{27}{21}, 21x = 189, x = 9$	$\frac{x}{7} = \frac{27}{21}, 21x = 189 = 9$	$\frac{x}{7} = \frac{27}{21}, x = 9$	$\frac{x}{7} = \frac{27}{21} = 9$	$\frac{x}{7} = \frac{27}{21} \text{ so } x = 9$	$\frac{x}{7} = \frac{27}{21} = x = 9$
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7.2.C

Use corresponding parts when solving problems involving similar figures.

When students are given similar figures, they sometimes fail to account for the orientation of the figures. For example:



When students are told triangle ABC and triangle DEF are similar and given lengths for AB, AC, and DE and asked to determine the length of FD, they solve the problem as if side AB corresponds to side FD because they are both vertical lines on the left-hand side of the triangle.

7.2.E

Have more practice writing equations to represent information in a table.

Students sometimes take the values in the table and connect them incorrectly to the corresponding variable. For example, when writing an equation to represent the relationship between time (t) and distance (d) in the table below, students sometimes write the equation “ $0.5t = 120d$ ” which does not represent the data in the table.

Time (hours)	0.5	1.0	1.5	2.0
Distance (feet)	120	240	360	480

Students can check the equation they write to verify that it represents the data in the table by substituting different values for the variables.

7.2.F

Understand slope is a numeric value.

Students can use whole numbers, fractions, and/or decimals to represent the slope of a line. However, when asked to determine the slope of a line students sometimes verbally describe the line, which is not the slope.

Slope	Not Slope
-3	Down 3, over 1
$\frac{2}{5}$	The line goes up 2 and then right 5
-1.5	Every time it goes to the left 2 it also goes up 3

Additionally, students sometimes include a variable along with the slope, e.g., students write “ $2x$ ” to represent the slope of a line with a slope of 2.

7.2.F

<p>Understand that slope represents a ratio between two changing values.</p>	<p>When determining the slope, students should use values from the vertical and horizontal scales. Teachers can help students work through the common misconception of counting the number of vertical and horizontal grid lines or spaces by providing students with graphs that use different scales on the vertical and horizontal axes.</p> <p>Example: Students sometimes write that the slope of the line in Tamyra’s Earnings graph is 3 because they are counting the grid lines or spaces (3 up and 1 over) rather than a slope of 5 which uses the values on the scale (15 up and 3 over).</p> <div style="text-align: center;"> </div>
<p>Have more practice with lines with negative slopes.</p>	<p>Students do better determining the slope of a line with a positive slope than the slope of a line with a negative slope. Frequently, students simply ignore the fact that the slope is negative and choose or write a positive value that represents the ratio of “rise” to “run.”</p>

7.3.D

<p>Use formulas to determine measurements of 2- and 3-dimensional figures.</p>	<p>Students often repeat or restate the formula for surface area or volume from the formula page; this earns no credit as students are not using the formulas. Students can earn credit for a narrative of the steps they took to determine the surface area of volume when they use specific values from the figure or situation.</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: left; padding: 5px;">Restating the Formula (no credit for work)</th> <th style="text-align: left; padding: 5px;">Narrative of steps (credit for work shown)</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;">I multiplied pi times the radius squared times the height</td> <td style="padding: 5px;">I multiplied 3.14 times 9 because the radius was 3 and then multiplied by the height of 16</td> </tr> <tr> <td style="padding: 5px;">I used $\pi r^2 h$ to find the volume</td> <td style="padding: 5px;">I used $\pi 3^2 (16)$ to find the volume</td> </tr> </tbody> </table>	Restating the Formula (no credit for work)	Narrative of steps (credit for work shown)	I multiplied pi times the radius squared times the height	I multiplied 3.14 times 9 because the radius was 3 and then multiplied by the height of 16	I used $\pi r^2 h$ to find the volume	I used $\pi 3^2 (16)$ to find the volume
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7.4.B

Understand how theoretical probability can be used to determine experimental outcomes.	<p>Students do well when asked to determine the theoretical probability of an event. However, students struggle to use that theoretical probability to determine the expected number of times that event will occur in a given number of trials. Teachers can help students by connecting expected outcomes to the proportional reasoning skills student also develop at this grade level.</p> <p>Example: The theoretical probability of rolling a 4 on a six-sided number cube is $\frac{1}{6}$. How many times would you expect to roll a 4 when you roll the number cube 6 times? When you roll it 12 times? When you roll it 18 times? When you roll it 60 times?</p>
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7.4.D

Understand how histograms represent a set of data without representing individual data points.	<p>Students struggle understanding that the intervals on the horizontal axis of a histogram place individual data points into groups. Students sometimes think that histograms, because they do not show individual data points from the set, do not accurately represent the data set. This is especially true when going from a list of data points to a histogram.</p> <p>Teachers can help students understand this organizational aspect of histograms by showing how other data displays, such as circle graphs, also do not represent individual data points but do accurately represent the data set.</p>
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Considerations for Grade 8:

To improve scores on the MSP, students should:

8.1.C

Reference specific parts of the graph of a linear function when writing a verbal description.	<p>Students are often too vague in their descriptions of the graph of a linear function, and make unclear statements such as:</p> <ul style="list-style-type: none">- The graph is negative- It's going positive- The line went to the right of the y-axis- The line is falling to the left <p>Students can clarify their verbal descriptions by including specific parts of the graph, in statements such as:</p> <ul style="list-style-type: none">- The slope of the line is negative- The x-intercept is negative- The line goes through the point (2, -1)- The line crosses the y-axis at -3
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8.1.E

Include information from the situation when explaining the meaning of the y-intercept and slope of the graph of a linear function.	<p>When asked to describe what the slope or y-intercept represents, students define slope or y-intercept rather than connect it to the situation; “the y-intercept is where the line crosses the vertical axis” does not describe what the y-intercept represents. “The y-intercept is how many newspapers Marianne had before she started her delivery” is an example of what the y-intercept might represent.</p> <p>When describing the slope, students should make reference to both changing values; i.e., “the average miles per hour” or “how many full turns the wheel made per second” or “the slope represents the cost for one pound of almonds.”</p>
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8.2.F

Use correct notation to represent the steps taken when using the Pythagorean Theorem.

Students frequently use incorrect notation to represent the steps they take, usually around squaring a number or taking the square root.

Sample Steps to a Problem	Correct Notation
1	$3.4^2 + b^2 = 17^2$
2	$11.56 + b^2 = 289$
3	$b^2 = 289 - 11.56$
4	$b^2 = 277.44$
5	$\sqrt{b^2} = \sqrt{277.44}$
6	$b \approx 16.7$

For step 5, taking the square root of both sides, students write incorrect statements such as:

$$b^2 = \sqrt{277.44}$$

$$\sqrt{b} = \sqrt{277.44}$$

$$\sqrt{b} = 277.44$$

$$\sqrt{b} \approx 16.7$$

Students occasionally skip steps and write incorrect statements and run-on equations like:

$$17^2 - 3.4^2 \approx 16.7$$

$$17^2 - 3.4^2 = \sqrt{277.44} \approx 16.7$$

$$289 - 11.56 = \sqrt{277.44}$$

$$289 - 11.56 = \sqrt{277.44} \approx 16.7$$

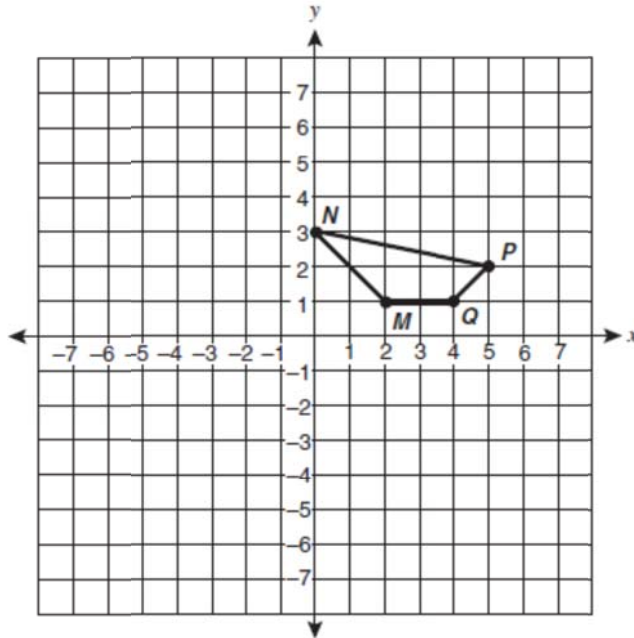
$$b^2 = \sqrt{277.44} \approx 16.7$$

This seems partially due to the student not understanding that taking the square root of a number is an operation and, unless done to both sides of an equation, results in a run-on equation.

8.2.G

Use the Pythagorean Theorem when determining the distance between two points on a coordinate plane.

Students need to understand that the distance between two points on a coordinate grid is a straight line. Commonly, students will simply count spaces or lines to get from one point to another. For example, when students are asked to determine the distance between point M and point P, students count 3 spaces over from M and then 1 space up to get to point P and state the distance is 4.



Teachers can help students identify that the two points are the end points of the hypotenuse of a right triangle and see that “counting over” and “counting up” gives the lengths of the legs of the triangle.

8.3.C

Understand a trend line is a single line that models data in a scatterplot.

Students sometimes draw multiple lines connecting the data points on a scatterplot rather than a single line; this gives the appearance of a line graph rather than a trend line.

Also, students sometimes draw a line so that all the data points are either above or below the line; this is not a trend line as it does not model the data.

8.3.F

Understand how to determine the probability of compound events.

While students do well determining the probability of each event in a compound situation, they struggle to understand how to use those probabilities to determine the compound probability of two events, both independent and dependent. Teachers can help students develop this understanding by building on the work done in 7th grade to represent the sample space of probability experiments.

Use specific language when describing different regions of a three-circle Venn Diagram.

Students will often describe a region as “between” two circles. It is unclear what it means to be “between” overlapping circles. Students seem to use “between” in two different ways: the first is as the region in the intersection of those two circles and not in the intersection of the third (Figure A) and the second is as the intersection of both circles (Figure B).

Students also frequently reference the “center” or “middle” of the diagram. This seems to imply the intersection of all the circles (Figure C) but is also unclear as sometimes students use it to refer to other intersecting regions such as shown in Figures A and B.

Students seem to have the greatest difficulty describing regions such as shown in Figure A that are in the intersection of two of the circles but not the third circle. They also struggle with describing a region that is within one of the circles but not in the intersection of either of the other circles. (Figure D).

Figure A

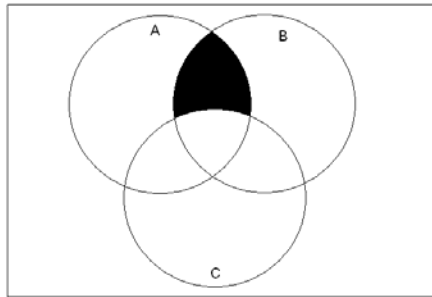


Figure C

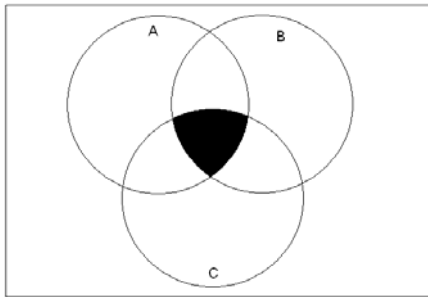


Figure B

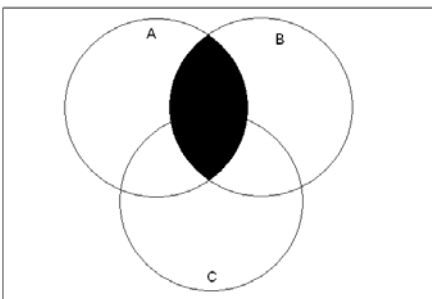
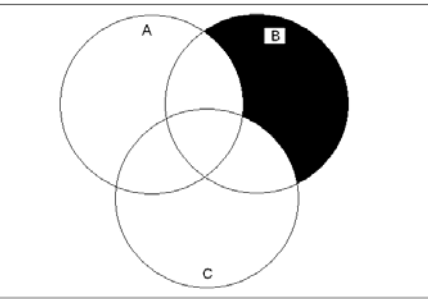


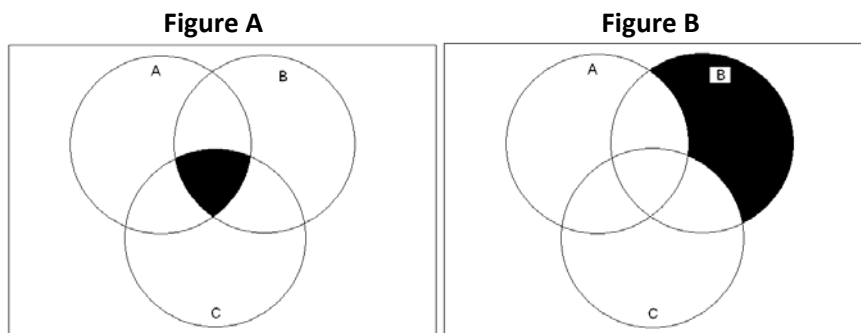
Figure D



8.3.G and 8.5.G

Have more practice identifying the meaning of numbers in different regions of a three-circle Venn Diagram.

Students often do not recognize that the intersection of all three circles represents a value that is common to all three circles, such as in Figure A. Additionally, they do not include the values from all the regions inside a circle when determining the total number of objects. They often just use the value from the region in the circle that is not in the intersection of either of the other two circles as the total for that circle, such as in Figure B.



8.4.A

Have more practice converting between standard notation and scientific notation.

The most common error students make when converting a number like 3.02×10^4 from scientific notation to standard notation is to add 4 zeros to the end of the number (because the exponent is 4) and then dropping the decimal point altogether to get the number 3,020,000. Teachers can help students understand the value of 10^4 and then connect it to the work students have done since 6th grade to multiply decimals. After that work is done, teachers can help students generalize rules to work with very large or very small exponents.