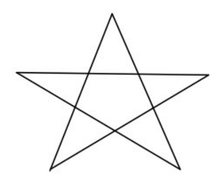
Star Polygons

Definition. An *{n/m} star polygon* is the shape formed by placing *n* dots equally spaced around a circle and connecting each one to those *m* spaces away. Don't worry about drawing them perfectly! Note the imperfectly drawn {5/2} star polygon below.

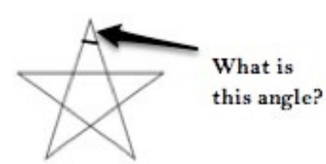
1. Trying drawing some star polygons:
   a. Draw a {5/3} star polygon.
   b. Draw a {7/3} star polygon and a {7/4} star polygon
   c. Draw an {8/3} star polygon.
   d. Draw a {12/5} star polygon.
   e. Draw a {4/2} star polygon. A {6/3} star polygon. What can you say about {*2n/n*} stars in general?
   f. Draw some others too. What else do you notice?



2. What does an {*n*/n} star look like?

3. What is a {n/1} star polygon?

4. Draw a {10/4} star polygon. Notice that it looks like 2 {5/2} stars. Which stars consist of a single drawn line, and which are multiple copies of other stars? If you decompose a {60/35} star polygon, what smaller stars is it made of, and what type of stars are they?



5. In a {5/2} star polygon, there are 5 intersections formed inside the star. In the {8/3} star polygon, there are 16 intersections. Find a formula for the number of intersections in an {*n*/*m*} star polygon.

6. What is the measurement of one of the angles of an {*n*/1} star polygon?

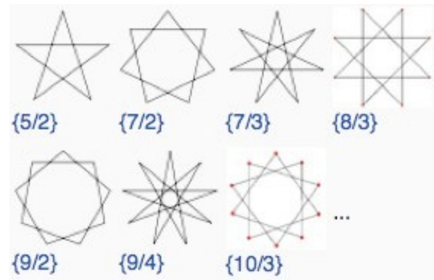
7. Find the angle of a {5/2} star polygon.

8. Find the angle of an {8/3} star polygon.

9. Find a formula for the angle of an {*n/m*} star polygon.

10. What {*n/m*} star polygon has an angle of 18 degrees exactly? Is there only one answer?

Star Polygon
Teacher's Notes



The overall goal of this activity is investigate star polygons, and especially to calculate the angles of star polygons. The drawings don't have to be perfect! However, rulers are a good idea.

Selected hints, answers, and solutions

2. An {n/1} star polygon is just a regular n-gon, e.g., a {3/1} star is an equilateral triangle.

3. If n and m have common factors, then the stars break up. Relative primeness is the key here.

4. A first attack on this problem is just to count, but smart counting is double counting. Consider how many lines there are in the star, how many times each line crosses other lines, and how many times you've counted the same intersection point if you count in this way. The final formula is n(m-1), which is quite clean.

5. An {n/1} star is just a regular n-gon. Break it up into n-2 triangles to get 180(n-2) as the sum of the degrees, and divide by n to get an individual angle.
   HINT 1: Finding angle sums of all n angles is the easier way to see the patterns in all of the rest of the problems.
   PROVOCATION: If you divide a regular n-gon like a pizza, you get n triangles. Why isn't the formula for the angle sum of an n-gon 180n?

6. One attempt, and its a fun one, is to start with the angles of the regular pentagon on the inside, and then work out. Don't forget to find the angle sum, which is 180 degrees (just like a triangle!).

8. There are many ways to approach the general problem. One I like is to imagine "walking" around the star. By the time you get back to where you started, you'll have made exactly *m* full turns (why?), so the sum of the exterior angles is 360m degrees.
   On the other hand, the exterior and interior angles are supplements, and there are n pairs of them. Subtracting gives the sum of the interior angles as 180n-360m degrees. This works even if the star is irregular.
   Making a chart of the angle sums can be a great way to get a conjecture for the formula when students are searching for it.