Pattern Blocks Angles

A *regular* n-gon has all its sides and angles congruent.
Our big question: which regular n-gons can you build with pattern blocks?

The problems below will help break this problem down.
A helpful fact you can use: the interior angles in any triangle add up to 180 degrees.

1. All the angles of the orange square are 90 degrees. Find all the other angles in all the pattern blocks.

2. Draw any four-sided shape (4-gon). What do its interior angles add up to?

   Now draw a 5-gon. What do its interior angles add up to?

   Now draw a 6-gon. What do its interior angles add up to?

   Now draw a 7-gon. What do its interior angles add up to?

3. What do the interior angles of any n-gon add up to?

4. A *regular* n-gon has all its sides and angles congruent. Find a formula for one interior angle of a regular n-gon.

5. **Which regular n-gons can you build with pattern blocks?** Build the ones that your formulas predict are possible to build. What is the largest n for which you can build a regular n-gon from pattern blocks?

Prepared for the Julia Robinson Festival by Daniel Finkel

Pattern Block Angles
Teacher's Notes

The overall goal is to understand how angles work, figure out a formula for the interior angles of regular n-gons, and use it to prove that only certain regular n-gons can be built from pattern blocks. This is actually a pretty powerful application: you prove that infinitely many shapes are impossible to build.

Selected hints, answers, and solutions

1. Hint: Try to arrange the same pattern block around a point. For example, six triangles fit around one point, which means their six angles must add up to 360 degrees, which means each one is 60 degrees. You can also compare angles of shapes to others you already know.
   Answer: All the shapes have angles that are multiples of 30 degrees.

2. Hint: try breaking the polygons up into triangles, and use what you know about the sums of angles in triangles.
   A good question to challenge the students: you can break a pentagon into five triangles with a point in the middle (i.e., the pizza cut). Why don't the angles of a pentagon add up to 5x180 degrees?
3. Answer: Any n-gon can be split into n-2 triangles without creating extra points, so the interior angles of an n-gon sum to $180(n-2)$.
4. Answer: If all the angles are the same, they must be the last answer divided by n, i.e., $180(n-2)/n$.
5. You know pattern blocks can only build a regular n-gon if that n-gon has angles that are a multiple of 30. Try subbing in values of n into the last formula and see what happens. What happens as n gets larger? (What's the geometric picture of this?)
   It turns out that the largest n for which you can build an n-gon with pattern blocks is $n=12$. There are many ways to build a regular n-gon. For example, put 12 tan rhombuses around a center point, and fill in the gaps with triangles.