

# Chess Attacks

Here's how chess pieces move: rooks move horizontally or vertically. Bishops move diagonally. Knights hop two spaces horizontal/vertical and one space vertical/horizontal. Queens move horizontally, vertically, or diagonally.

1. (Warm-up) Place eight rooks on a chessboard without any of them attacking each other. How do you know that placing more than eight with this condition is impossible?
2. What is the maximum number of knights you can place on a chessboard without any of them attacking each other?
3. What is the maximum number of bishops you can place on a chessboard without any of them attacking each other?
4. Now consider these questions for an  $n$  by  $n$  board. Can you find formulas in terms of  $n$  for the max number of rooks, knights, bishops, and/or queens you can place on a chessboard?
5. A *torus chessboard* is one where the left side connects to the right and the top connects to the bottom. How many squares does a bishop attack on a torus chessboard?
6. For which  $n$  by  $m$  torus chessboards does a single bishop attack every square?
7. For any  $n$  by  $m$  torus chessboard, what is the maximum number of rooks you can place without any of them attacking each other?
8. For any  $n$  by  $m$  torus chessboard, what is the maximum number of knights you can place without any of them attacking each other?
9. For any  $n$  by  $m$  torus chessboard, what is the maximum number of bishops you can place without any of them attacking each other?
10. For any  $n$  by  $m$  torus chessboard, what is the maximum number of queens you can place without any of them attacking each other?

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## Teacher's Notes

It's useful to pose the following questions, especially when kids have trouble getting started:

- Did you try working on a smaller board to start?
- What happens on a 4 by 4 board, or a 5 by 5 board?

### **Selected hints, answers, and solutions**

1-5: The limiting factor is usually about the rows, columns or diagonals. For rooks, no two can be on the same row, so the max is 8. For bishops, no two can be on the same diagonal, so the max is 15 (and in fact, you can argue a bit more to show it's 14). Since knights only attack the opposite colored square, you can put them on the 32 white squares and they won't attack each other. Generalizing shouldn't be too bad for these.

6-7. These are neat questions to think about. In general, if  $n$  and  $m$  are relatively prime the bishop will hit all the squares on the board. Exploring based on smaller boards until you start to see what's going on may be the best approach.

8-10. These will be a matter of determining how many rows, columns, and diagonals there are on a torus board. Picking easy choices (like  $n=m$ , or  $n=2$ ) will help students gain some traction into this problem.