Trigonometric Functions

1. Suppose θ is an acute angle. Then $sin^{2}\left(θ\right)+cos^{2}\left(θ\right)=1$.

Six statements that outline an argument that proves this claim are shown in the table.

Order the statements 1 (first) through 6 (last) into a logical sequence to outline an argument that proves this claim.

| **Statement** | **Order (1 is first, 6 is last)** |
| --- | --- |
| Construct a right triangle that includes θ as one of its acute angles. |  |
| So $sin^{2}\left(θ\right)+cos^{2}\left(θ\right)=1$. |  |
| Give the label *a* to the side that is adjacent to θ, the label *b* to the side that is opposite θ, and the label *c* to the hypotenuse. |  |
| $\frac{a^{2}}{c^{2}}+\frac{b^{2}}{c^{2}}=1$ if we divide both sides by $c^{2}$. |  |
| $a^{2}+b^{2}=c^{2}$by the Pythagorean Theorem. |  |
| $\sin((θ))=\frac{b}{c}$ and $\cos((θ))=\frac{a}{c}$ by the definition of $\sin((θ))$ and $\cos((θ))$. |  |

1. Suppose that $\cos((θ))=\frac{2}{5}$ and that θ is in the 4th quadrant.

***a.*** What is the exact value of $\sin((θ))$?
***b.*** What is the exact value of $\tan((θ))$?
2. For an acute angle θ, sin(θ) can be defined in terms of the side-lengths of a right triangle that includes angle θ. Here is the definition and diagram:

Given a right-triangle with side-lengths *a* and *b* and hypotenuse *c*, if θ is the angle opposite *b*, then sin(θ) =$\frac{b}{c}$.



**Part A**

In the figure below, angle θ has a vertex at the origin, its initial side corresponds to the positive *x*-axis, and the terminal side intersects the unit circle at the point (*a*, *b*).



What is sin(θ) in terms of *a* and *b* according to the definition given?

 **Part B**

For angles that are not acute, the definition of sin(θ) is given in terms of the unit circle:

If angle θ has a vertex at the origin, its initial side corresponds to the positive *x*-axis, and the terminal side intersects the unit circle at the point (*a*, *b*), then sin(θ) = *b*.

In the figure shown below, what is sin(θ)?



1. The organizers of a community fair set up a small Ferris wheel for young children. The table shows the heights of one of the cars above ground for different rotations of the wheel.

| **Angle of Rotation (radians)** | **Height Above the Ground (feet)** |
| --- | --- |
| 0 | 1 |
| $$\frac{π}{2}$$ | 7 |
| $$π$$ | 13 |
| $$\frac{3π}{2}$$ | 7 |
| $$2π$$ | 1 |
| $$\frac{5π}{2}$$ | 7 |
| $$3π$$ | 13 |
| $$\frac{7π}{2}$$ | 7 |
| $$4π$$ | 1 |

**Part A**The function$h\left(x\right)=a\sin((x-\frac{π}{2}))+b$, where *a* and *b* are constants, models the height of the
Ferris wheel car at a rotation of *x* radians. What is the values of *a* and *b*?

A. *a* = 1 and *b* = 12
B. *a* = 6 and *b* = 7
C. *a* = 7 and *b* = 6
D. *a* = 12 and *b* = 1

**Part B**Consider the graph of $y=h(x)$ in the coordinate plane. Which statements are true?
Select **all** that apply.

A. The amplitude of the graph is 12.
B. The period of the graph is $2π$.
C. The midline of the graph is at *y* = 13.
D. The graph is increasing for $4π<x<5π$.
E. The graph is decreasing for $\frac{11π}{2}<x<\frac{13π}{2}$.
F. The graph has a maximum at *y* = 13.

1. The unit circle is shown.



Use the unit circle and the indicated triangle to answer these questions.
***a.*** What is the exact value of the sine of $\frac{π}{6}$?
***b.*** What is the exact value of the cosine of $\frac{π}{6}$?
2. A wheel with a radius of 0.2 meters begins to move along a flat surface so that the center of the wheel moves forward at a constant speed of 2.4 meters per second. At the moment the wheel begins to turn, a marked point *P* on the wheel is touching the flat surface.



The function *y* gives the height, in meters, of the point *P*, measured from the flat surface, as a function of *t*, the number of seconds after the wheel begins moving.

Sketch a graph of the function *y* for *t* ≥ 0. Interpret two characteristics of your graph (minimum, maximum, midline, period, amplitude, shift, etc.) with respect to the real-world context.

**Teacher Material**

A-SSE.A

Interpret the structure of expressions.

A-CED.A

Create equations that describe numbers or relationships.

F-IF.A

Understand the concept of a function and use function notation.

F-IF.B

Interpret functions that arise in applications in terms of a context.

F-IF.C

Analyze functions using different representations.

| **Question** | **Claim** | **Key/Suggested Rubric** |
| --- | --- | --- |
| 1[[1]](#footnote-1) | 3 | **1 point:**

| **Statement** | **Order** **(1 is first, 6 is last)** |
| --- | --- |
| Construct a right triangle that includes θ as one of its acute angles. | 1 |
| So $sin^{2}\left(θ\right)+cos^{2}\left(θ\right)=1$. | 6 |
| Give the label *a* to the side that is adjacent to θ, the label *b* to the side that is opposite θ, and the label *c* to the hypotenuse. | 2 |
| $\frac{a^{2}}{c^{2}}+\frac{b^{2}}{c^{2}}=1$ if we divide both sides by $c^{2}$. | 4 or 5 |
| $a^{2}+b^{2}=c^{2}$by the Pythagorean Theorem. | 3 or 4 |
| $\sin((θ))=\frac{b}{c}$ and $\cos((θ))=\frac{a}{c}$ by the definition of $\sin((θ))$ and $\cos((θ))$. | 3 or 5 |

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| 2[[2]](#footnote-2) | 2 | **2 points:** $-\frac{\sqrt{21}}{5}$, or equivalent AND$-\frac{\sqrt{21}}{2}$, or equivalent.**1 point:**$ -\frac{\sqrt{21}}{5}$, or equivalent OR$-\frac{\sqrt{21}}{2}$, or equivalent. |
| 3[[3]](#footnote-3) | 3 | **2 points:** *b*, or equivalent AND $\frac{1}{3}$, or equivalent.**1 point:** *b*, or equivalent AND $\frac{1}{3}$, or equivalent. |
| 4[[4]](#footnote-4) | 2 | **2 points:** Selects B AND Selects A, B, D, E, and F**1 point:** Selects B OR Selects A, B, D, E, and F |
| 5[[5]](#footnote-5) | 2 | **2 points:**$ \frac{1}{2}$, or equivalent AND$ \frac{\sqrt{3}}{2}$, or equivalent.**1 point:**$ \frac{1}{2}$, or equivalent OR$ \frac{\sqrt{3}}{2}$, or equivalent. |
| 6[[6]](#footnote-6) | 4 | **1 point:** Answers will vary. **Example:** The graph has a minimum value of 0 which means point *P* is on the flat surface, and never goes below the flat surface. The graph has a midline of *­y* ­= 0.2 because the radius of the wheel is 0.2 meters and point *P* has a height from 0 (on the flat surface) to 0.4 (the highest point possible). |

1. Adapted from Smarterbalanced.org. Grades 11, Claim 3 Item Specifications. Internet. Available from <http://www.smarterbalanced.org/smarter-balanced-assessments/>; accessed 11/2015. [↑](#footnote-ref-1)
2. <https://www.illustrativemathematics.org/content-standards/HSF/TF/C/8/tasks/1835> [accessed on November 1](https://www.illustrativemathematics.org/content-standards/HSG/GMD/A/3/tasks/514%20accessed%20on%20November%201), 2015, is licensed by [Illustrative Mathematics](https://www.illustrativemathematics.org/) under [CC BY-NC-SA 4.0](http://creativecommons.org/licenses/by-nc-sa/4.0/). [↑](#footnote-ref-2)
3. Adapted from Smarterbalanced.org. Grades 11, Claim 3 Item Specifications. Internet. Available from <http://www.smarterbalanced.org/smarter-balanced-assessments/>; accessed 11/2015. [↑](#footnote-ref-3)
4. <https://prc.parcconline.org/system/files/Algebra%202%20-%20EOY%20-%20Item%20Set.pdf> accessed on November 1, 2015, Copyright© 2015 PARCC Inc. All Rights Reserved. PARCC® is a registered trademark of PARCC Inc. [↑](#footnote-ref-4)
5. <https://www.illustrativemathematics.org/content-standards/HSF/TF/A/3/tasks/1898> [accessed on November 1](https://www.illustrativemathematics.org/content-standards/HSG/GMD/A/3/tasks/514%20accessed%20on%20November%201), 2015, is licensed by [Illustrative Mathematics](https://www.illustrativemathematics.org/) under [CC BY-NC-SA 4.0](http://creativecommons.org/licenses/by-nc-sa/4.0/). [↑](#footnote-ref-5)
6. <https://www.illustrativemathematics.org/content-standards/HSF/TF/B/5/tasks/595> [accessed on November 1](https://www.illustrativemathematics.org/content-standards/HSG/GMD/A/3/tasks/514%20accessed%20on%20November%201), 2015, is licensed by [Illustrative Mathematics](https://www.illustrativemathematics.org/) under [CC BY-NC-SA 4.0](http://creativecommons.org/licenses/by-nc-sa/4.0/). [↑](#footnote-ref-6)